International Journal of Innovative Research and Practices Vol.2, Issue 9, September 2014
ISSN 2321-2926

Assignment Technique for solving Transportation Problem – A Case Study

N. Santosh Ranganath

Faculty Member, Department of Commerce and Management Studies
Dr. B. R. Ambedkar University, Srikakulam, Andhra Pradesh, India

Introduction:

Transportation and assignment models are special purpose algorithms of the linear programming. The simplex method of Linear Programming Problems(LPP) proves to be inefficient is certain situations like determining optimum assignment of jobs to persons, supply of materials from several supply points to several destinations and the like. More effective solution models have been evolved and these are called assignment and transportation models.

The transportation model is concerned with selecting the routes between supply and demand points in order to minimize costs of transportation subject to constraints of supply at any supply point and demand at any demand point. Assume a company has 4 manufacturing plants with different capacity levels, and 5 regional distribution centres. 4 x 5 = 20 routes are possible. Given the transportation costs per load of each of 20 routes between the manufacturing (supply) plants and the regional distribution (demand) centres, and supply and demand constraints, how many loads can be transported through different routes so as to minimize transportation costs? The answer to this question is obtained easily through the transportation algorithm.

Similarly, how are we to assign different jobs to different persons/machines, given cost of job completion for each pair of job machine/person? The objective is minimizing total cost. This is best solved through assignment algorithm.

Uses of Transportation and Assignment Models in Decision Making

The broad purposes of Transportation and Assignment models in LPP are just mentioned above. Now we have just enumerated the different situations where we can make use of these models.

Transportation model is used in the following:

- To decide the transportation of new materials from various centres to different manufacturing plants. In the case of multi-plant company this is highly useful.
- To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful.
- To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful. These two are the uses of transportation model. The objective is minimizing transportation cost.

Assignment model is used in the following:

- To decide the assignment of jobs to persons/machines, the assignment model is used.
- To decide the route a traveling executive has to adopt (dealing with the order inn which he/she has to visit different places).
- To decide the order in which different activities performed on one and the same facility be taken up.

In the case of transportation model, the supply quantity may be less or more than the demand. Similarly the assignment model, the number of jobs may be equal to, less or more than the number of machines/persons available. In all these cases the simplex method of LPP can be adopted, but transportation and assignment models are more effective, less time consuming and easier than the LPP.

Table 1

The cost matrix						
Factory	1	2	3	4	Maximum capacity	Cost of production (Rs)
A	7	5	6	4	10	10
B	3	5	4	2	15	15
C	4	6	4	5	20	16
D	8	7	6	5	15	15
Monthly requirements	8	12	18	22		
Sale price (Rs)	20	22	25	18		

Compared to the simplex method, assignment and transportation techniques are special purpose algorithms which are useful for solving some types of linear programming problems. The assignment problem itself is a special case of the transportation problem.

Usually for the initial allocation in the case of a transportation problem, methods such as north-west corner rule, Vogel's approximation method and least cost entry method are used. For the purpose of optimality the MODI check is carried out finally.

A specific condition is that the number of allocations should always equal m+n-1, where m=number of rows and n=number of columns.

An interesting feature of the transportation problem arises in the case of initial allocation through the assignment technique. Consider a situation where the data constitute selling price, cost of production and shipping costs. If the objective is to maximise it can be shown that each data can be construed separately as a transportation problem — that is, maximise selling price, minimize cost of production and minimise transportation costs.

If the selling price and the cost of production are uniform to each column and row respectively, and the transportation costs are different it can be further shown that the solution obtained for the allocation of the transportation cost individually will also be the solution for the selling price as well as the cost of production, taken independently or collectively, that is, selling price minus cost of production.

This is because of the fact that application of the assignment technique in case of selling price minus cost of production results in a constant difference and ultimately will lead to zero in the case of each cell. If all the cells are zero there are any number of solutions.

A problem to solve

A company has four factories situated in four different locations in the country and four sales agencies located in four other locations. The cost of production (rupees per unit), the sales price (rupees per unit) and shipping costs (rupees per unit) in the cells of the matrix, and monthly capacities and monthly requirements are given in Table 1. Find the monthly production and distribution schedule, which will maximise profit.

In this problem, the usual procedure is to compute the profit for each of the sales agency corresponding to the factory.

Table 2

Optimal solution						
Factory	Sales agency	Quantity in units	Profit per unit (Rs)	Total profit (Rs)	Transportation cost per unit (Rs)	Total transportation cost (Rs)
A	2	10	7	70	5	50
B	4	15	1	15	2	30
C	1	8	0	0	4	32
C	3	12	5	60	4	48
D	2	2	0	0	7	14
D	3	6	4	24	6	36
D	4	7	-2	-14	5	35
Total		60		155		245

For example, for Factory A and sales agency 1, the profit will be Rs 20 -(10+7) = Rs 3. By applying Vogel's approximation method and MODI check, the optimal solution (Chart 2) is obtained. It may be noticed that a minimum of two iterations are necessary to arrive at the optimal solution.

Table 3

Final cost matrix											
Initial matrix				Row operation				Column operation			
7	5	6	4	3	1	2	0	3	0	2	0
3	5	4	2	1	3	2	0	1	2	2	0
4	6	4	5	0	2	0	1	0	1	0	1
8	7	6	5	3	2	1	0	3	1	1	0

As already stated, the optimal allocation is found only with respect to the transportation cost. Applying the assignment technique, the final cost matrix is obtained (Chart 3).

After the column operation, it can be noticed that element zero is found in six places. Out of which, cells R1C2, R2C4, R3C1, R3C3 and R4C4 can be allocated with the quantity 10, 15, 8, 12 and 7 respectively. To satisfy the total condition of m+n-1, it is necessary to allocate in two more cells, with the next least cost.

In this case the least cost is one which could be found in cells R4C2 and R4C3 which requires to be allocated. The balance quantity of two as well as six will be allocated in these cells.

This solution will obviously be optimal since the allocation happens to be through the element zero and the next least cost one, which is normally the basic property of any assignment model.

As already seen, the solution is optimal, with the transportation cost being Rs 245.

Simple accounting equation suggests that the net profit equals selling price minus cost. This aspect can be seen in the above problem. Since the selling price remains constant for each sales agency individually, the cost of production constant for each factory, the total sales equals Rs 1,270 (Rs 20 x 8 + 22 x 12 + 25 x 18 + 18 x 22) and the cost of production equals Rs 870 (Rs 10 x 10 + 15 x 15 + 20 x 16 + 15 x 15). The transportation cost is Rs 245 and the net profit equals Rs 155, which was the solution obtained initially.

Application of assignment technique results in the elimination of a constant figure from each of the rows as well as columns. This could be used as a general case for all transportation problems.

A major advantage is that the initial allocation can be easily carried out in the cells having zero as the element. If all the m+n-1 allocations are possible through zero element it can be easily concluded that such a solution is always optimal. However such a situation is quite rare. In any case after allocating the resources in the zero cell, if the balance allocation is carried out in the next least cost apart from zero, one can still be towards the optimal solution. Almost all transportation problems whether balanced or unbalanced can be solved by using the assignment method and, in majority of the cases, the solution is obtained in lesser number of iterations.

References

Hamdy A.Taha.” Operations Research: An Introduction “,Prentice Hall, 7 editions 5, USA,2006.

John D. McGregor, David A. Sykes “A Practical Guide to Testing Object-Oriented Software”, Addison-Wesley Professional, USA, 2001.

Peretz Shovel.” functional and object-oriented analysis and design “(an integrated methodology), Idea Group Publishing (an imprint of Idea Group Inc.), United States of America, 2006.

Prem Kumar Gupta, D.S.Hira.” Operations Research”, An Introduction, S.Chand and Co., Ltd. New Delhi, 1999.

Beale, E. M. L., "An Algorithm for Solving the Transportation Problem when the Shipping Cost Over Each Route is Convex", Naval Research Logistics Quarterly, Vol. 6 (1959), pp. 43-56.

Dantzig, G. B., Ford, L. R., JR. and Fulkerson, D. R., "A Primal-Dual Algorithm for Linear Programs", in Linear Inequalities and Related Systems (Kuhn and Tucker, eds.), Annals of Mathematics Study No. 38, Princeton University Press, Princeton, 1956.

Flood, Merrill M., "The Travelling Salesman Problem", Operations Research, Vol. 4 (1956), pp. 61-75.

Ford, l. R., Jr. and Fulkerson, D. R., "A Simple Algorithm for Finding Maximal Network Flows and an Application to the Hitchcock Problem", Canadian Journal of Mathematics, Vol. 9 (1957), pp. 210-218

Fulkerson, D. R., "An Out-of-Kilter Method for Minimal Cost Flow Problems", Journal of the Society for Industrial and Applied Mathematics, Vol. 9 (1961), pp. 18-27.

Gleyzal, A., "An Algorithm for Solving the Transportation Problem", Journal of Research of the National Bureau of Standards, Vol. 54, No. 4, April 1955, pp. 213-216.

Kuhn, H. W., "The Hungarian Method for the Assignment Problem", Naval Research Logistics Quarterly, Vol. 2 (1955), pp. 83-97.

Munkres, James, "Algorithms for the Assignment and Transportation Problems", Journal of the Society for Industrial and Applied Mathematics, Vol. 5 (1957), pp. 32-38