On Some Fixed Point Theorems in Dislocated Quasi b-Metric Spaces

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Abstract: *In this paper, we establish the two fixed point theorems in dislocated quasi b-metric spaces. Incidentally we have obtained the results of Sastry [12] and Aage [13] as corollaries.*

Keywords: *S-metric Space, quasi metric spaces, quasi-b-metric spaces, dislocated quasi-b-metric spaces, Banach contraction.*

Classification: *54H25, 47H10.*

1. Introduction

The concept of metric space was introduced Frechet [1] in 1906.The concept of *b*-metric space was introduced by Bakhtin [2] and was used by Czerwik [3] to study contraction mapping in *b*-metric space. The concept of dislocated quasi-*b*-metric space is introduced by F.M.Zeyada [4] and proved some fixed theorems on it.For more works we refer [14,15,16,17,18]. Several researchers worked on generalized metric spaces andproved some fixed point theorems on generalized metric spaces [19,20,21,22]. In this paper, we present some new fixed point theorems in dislocated quasi-b-metric spaces.

2. Preliminaries

Definition 2.1(Zeyada[4]): A metric on a non empty set X is a function $D: X \times X \to [0,\infty)$ such that for all $x, y, z \in X$, the following conditions hold:

(i) $D(x,y) = 0 = D(y,x) \Rightarrow x = y,$ for all $x, y \in X$

(ii) $D(x,y) = D(y,x),$ for all $x, y \in X$

(iii) $D(x,y) \leq D(x,z) + D(z,y),$ for all $x, y, z \in X.$

The pair (X, D) is called dislocated metric space.

Definition 2.2(Zeyada [4]): A quasi metric on a non empty set X is a function $D: X \times X \to [0,\infty)$ such that for all $x, y, z \in X$, the following conditions hold:

(i) $D(x,x) = 0$ for all $x, y \in X$

(ii) $D(x,y) = 0 = D(y,x), \Rightarrow x = y,$ for all $x, y \in X$

(iii) $D(x,y) \leq D(x,z) + D(z,y),$ for all $x, y, z \in X.$

The pair $(X, D)$is called quasi-metric space.

Definition 2.3(Shah [5]): Let X be a non empty set. Let $D: X \times X \to [0,\infty)$ be a mapping and $k \geq 1$be a constant such that:

(i) $D(x,y) = 0 = D(y,x), \Rightarrow x = y,$ for all $x, y \in X$

(ii) $D(x,y) \leq k(D(x,z) + D(z,y)),$ for all $x, y, z \in X.$

The pair (X, D) is called quasi-*b*-metric space.

Definition 2.4(Alghamdi [6]): Let X be a non empty set. Let $D: X \times X \to [0,\infty)$ be a mapping and $k \geq 1$ be a constant such that:

(i) $D(x,y) = 0 \Rightarrow x = y,$ for all $x, y \in X$

(ii) $D(x,y) = D(y,x),$ for all $x, y \in X$

(iii) $D(x,y) \leq k(D(x,z) + D(z,y))$, for all $x, y, z \in X$.

The pair (X, D) is called *b*-metric like space.

Definition 2.5(Chakkrid [7]) Let X be a non empty set. Let $D: X \times X \to [0,\infty)$ be a mapping and $k \geq 1$ be a constant such that:

(i) $D(x,y) = 0 = D(y,x)$, $\Rightarrow x = y$, for all $x, y \in X$

(ii) $D(x,y) \leq k(D(x,z) + D(z,y))$, for all $x, y, z \in X$.

The pair (X, D) is called dislocated quasi-*b*-metric space or in short *dqb*-metric space.

Note2.6: The constant k is called coefficient of (X, D). It is clear that b-metric spaces, quasi-*b*-metric spaces and *b*-metric like spaces are *dqb*-metric spaces but converse is not true.

Example 2.7: Let $X = \mathbb{R}^+$ and for $p > 1$, $D: X \times X \to [0,\infty)$ defined by $D(x,y) = |x-y|^p + |x|^p$ for all $x, y, z \in X$. The (X, D) n is *dqb*-metric space with $k = 2^p$. But (X, D) is not b-metric space and also not dislocated quasi metric space.

Example2.8 : Let $X = \mathbb{R}^+$ and for p>1, $D: X \times X \to [0,\infty)$ defined by $D(x,y) = |2x-y|^2 + |2x+y|^2$ for all $x, y, z \in X$. Then (X, D) is *dqb*-metric space with $k = 2$. But (X, D) is not *b*-metric space and also not dislocated quasi metric space.

Definition2.9 (Chakkrid[7]) Let (X, D) be a *dqb*-metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. We say that $\{x_n\}$ is *dqb*-converges to x if $\lim_{n\to\infty} D(x_n, x) = 0 = \lim_{n\to\infty} D(x, x_n)$. Here x is called a *dqb*-limit of $\{x_n\}$ and written as $\{x_n\} \to x$.

Definition 2.10 (Chakkrid [7]) Let (X, D) be a dqb-metric spaces. Let $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is *dqb*-Cauchy sequence if $\lim_{n,m\to\infty} D(x_n, x_m) = 0 = \lim_{n,m\to\infty} D(x_m, x_n)$.

Definition 2.11 (Chakkrid [7]) A *dqb*-metric spaces (X, D) is said to be *dqb*-complete if every *dqb*-Cauchy sequence in it is *dqb*-convergent in X.

Lemma2.12 (Chakkrid [7]) Every subsequence of a *dqb*-convergent sequence in a *dqb*-metric space (X, D) is a *dqb*-convergent sequence.

Lemma2.13 (Chakkrid [7]) Every subsequence of a *dqb*-Cauchy sequence in a *dqb*-metric space (X, D) is a *dqb*-convergent sequence.

Lemma2.14 (Chakkrid [7]) If (X, D) is a *dqb*-metric space then a function $f: X \to X$ is continuous if and only if $x_n \to x$ then $fx_n \to fx$.

Lemma2.15 (Chakkrid [7]) Limit of a *dqb*-convergent sequence in *dqb*-metric space is unique.

Lemma2.16 (Chakkrid [7]): Let (X, D) be a *dqb*-metric space and $\{x_n\}$ be a sequence in it such that, $D(x_n, x_{n+1}) \leq \alpha D(x_{n-1}, x_n)$, $n = 1,2,3\ldots$ and $0 \leq \alpha < 1, \alpha \in [0,1)$ then $\{x_n\}$ is a Cauchy sequence in X.

Lemma2.17 (Chakkrid [7]): If x is a limit point of some *dqb*-convergent sequence in a *dqb*-metric space (X, D) then $D(x, x) = 0$

Lemma2.18 (Chakkrid [7]): Every *dqb*-convergent sequence in a *dqb*-metric space (X, D) is *dqb*-Cauchy sequence.

Definition2.19 (Jungck[8]) Let f and g be self maps of a set X. If $w=fx=gx$ for some $x \in X$, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition2.20(Jungck [8]) Let f and g be self maps of a set X. Then f and g are said to be weakly compatible if they commute at their coincidence point.

Lemma2.21 (Abbas [9]): Let f and g are weakly compatible self maps of a set X. If f and g have unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g.

Definition2.22(Chakkrid[7]) Let $f: X \to X$ be a self mapping of a set X, f is said to be sub sequentially convergent if for every sequence $\{x_n\}$ if fx_n is *dqb*-convergent then $\{x_n\}$ has a *dqb*-convergent subsequence in X.

Definition2.23(Chakkrid [7]) Let $f: X \to X$ be a self mapping of a set X, f is said to be sequentially convergent if for every sequence $\{x_n\}$ if fx_n is *dqb*-convergent then $\{x_n\}$ is also *dqb*-convergent in X.

Definition2.24 (Samet [10]) Let T be a self on a set X, and $\alpha: X \times X \to [0,\infty)$ be a function. We say that T is α-admissible mapping, if $x, y \in X$ then$\alpha(x,y) \geq 1 \Longrightarrow \alpha(Tx,Ty) \geq 1$

Definition2.25 (Jose [11]) Let (X,D) be a *dqb*-metric space and $T,S: X \to X$ be two mappings then the mapping$S$ is called T-Banach contraction if $\exists \alpha \in [0,1) \ni D(TSx,TSy) \leq \alpha\, D(Tx,Ty) for\ all\ x,y \in X$

Definition 2.26 (Jose [11]) Let (X,D) be a *dqb*-metric space and $T,S: X \to X$ be two mappings then the mapping S is called T-Kannan contraction if $\exists \alpha \in [0,\frac{1}{2})$such that

$$D(TSx,TSy) \leq \alpha[D(Tx,TSx) + D(Ty,TSy)], for\ all\ x,y \in X$$

Definition 2.27(Sastry [12])**:** Let (X,D)be a *dqb*-metric space and$T,S: X \to X$ be two mappings then the mapping S is called$T\phi$ $-$contraction if there exists altering distance function ϕ such that $D(TSx,TSy) \leq \phi\, D(Tx,Ty) for\ all\ x,y \in X$.

3. Main Results

Theorem 3.1(Sastry [12])Let (X,D) be a dqb-complete metric space and $f,g: X \to X$ be self mappings satisfying the inequality $D(fx,fy) \leq \phi\big(D(gx,gy)\big),\ for\ all\ x,y \in X$ where ϕ is altering distance function and $\phi\ (t) < \frac{t}{t}$if$t > 0, k > 1$. If $f(X) \subseteq g(X)$ and $g(X)$ is dqb-complete subspace of X then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.

Corollary 3.2(Aage [13]):et (X,D) be a dqb-complete metric space and $f,g: X \to X$ be self mappings satisfying the inequality $D(fx,fy) \leq \alpha\big(D(gx,gy)\big),\ for\ all\ x,y \in X$ where $\alpha \in [0,1)$ such that $\alpha k \leq 1$ and k is coefficient of (X,D).If $f(X) \subseteq g(X)$ and $g(X)$ is dqb-complete subspace of X, then f and g have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and g have common unique fixed point in X.

Lemma 3.3(Sastry [12])If $x_n \to x$ then $D(x,x) = 0$.

Proof: $D(x,x) \leq kD(x,x_n) + kD(x_n,x) \longrightarrow 0$

Therefore $D(x,x) = 0$.

Now we state and prove our main results

Theorem 3.4: Let (X,D) be a dqb-complete metric space and $f,g: X \to X$ be self mappings satisfying the inequality $D(fx,fy) \leq \phi(max\{D(fx,gx), D(fy,gy)\}), for\ all\ x,y \in X$ where ϕ is altering distance function and $\phi\ (t) < \frac{t}{k}, if\ t > 0, k > 1$. If $f(X) \subseteq g(X)$ and $g(X)$ is *dqb*-complete subspace of X, then f and $g$have unique point of coincidence in X. In addition if f and g are weakly compatible, then f and $g$have common unique fixed point in X.

Proof:

Let x_0 be any arbitrary point in X As $f(X) \subseteq g(X)$

We can choose that $x_1 \in X \ni fx_0 = gx_1$

Again we can choose $x_2 \in X \ni fx_1 = fx_2$

Repeating in the same manner, for $x_n \in X$

We can choose $x_{n+1} \in X \ni fx_n = gx_{n+1}\ for\ n = 0,1,2.....$

Now consider $D(gx_{n+1},gx_n) = D(fx_n,fx_{n-1}) \leq \phi(max\{D(fx_n,gx_n) + D(fx_{n-1},gx_{n-1})\}$

$$= \phi(max\{D(gx_{n+1},gx_n) + D(gx_n,gx_{n-1})\})$$

$$< \frac{1}{k} max\{D(gx_{n+1},gx_n) + D(gx_n,gx_{n-1})\}$$

Therefore $k\, D(gx_{n+1},gx_n) \leq max\{D(gx_{n+1},gx_n) + D(gx_n,gx_{n-1})\}$

If $D(gx_{n+1},gx_n) > D(gx_n,g_{n-1})$

Then $k\, D(gx_{n+1},gx_n) \leq D(gx_{n+1},gx_n)$

Which is a contradiction?

So $D(gx_{n+1},gx_n) < D(gx_n,g_{n-1})$

$k\, D(gx_{n+1},gx_n) \leq D(gx_n,gx_{n-1})$

$$D(gx_{n+1}, gx_n) \leq \frac{1}{k} D(gx_n, gx_{n-1})$$

By lemma 4.3.3, $\{x_n\}$ is dqb-Cauchy sequence in X.

Since *g(X)* is dqb-complete subspace of *X*.

So $\exists\, v \in g(X) \ni gx_n \longrightarrow v \; as \; n \longrightarrow \infty$

Since $v \in g(X)$ we can find $u \in X \ni gu = v$.

Now $D(gx_n, fu) = D(fx_{n-1}, fu)$

$$\leq \phi(max\,\{D(fx_{n-1}, gx_{n-1}) + D(fu, gu)\})$$

$$= \phi(max\,\{D(gx_n, gx_{n-1}) + D(fu, gu)\})$$

Therefore $D(v, fu) \leq \phi(max\{D(v, v) + D(fu, v)\})$

$$\leq \phi(D(fu, v)\})$$

$$D(v, fu) < \frac{1}{k} D(fu, v)$$

Similarly $D(fu, v) < \frac{1}{k} D(v, fu)$

Therefore $D(v, fu) < \frac{1}{k} D(fu, v) \leq \frac{1}{k^2} D(v, fu)$

Which is a contradiction always.

Therefore $D(v, fu) = 0$ and $D(fu, v) = 0$

$$fu = v$$

$gu = v$

$fu = gu = v$

This shows that u is a point of coincidence of f and g in X

Now we claim that this point of coincidence of f and g in X is unique.\\

On the contrary we assume that there exist $w \in X$ such that $fw = gw$

Now $D(gu, gw) = D(fu, fw) \leq \phi(max\{D(fu, gu), D(fw, gw)\})$

$$\leq \phi(max\{D(u, u), D(fw, gw)\})$$

$$\leq \phi(D(fw, gw))$$

$$\leq \frac{1}{k} D(fw, gw)$$

$$\leq \frac{1}{k} [kD(fw, fu) + kD(fu, gw))$$

Therefore $D(gu, gw) = D(fw, fu) + D(fu, gw)$

$$D(fw, fu) = 0$$

Therefore$D(gu, gw) = 0$

Similarly $D(gw, gu) = 0$

Thus $D(gu, gw) = D(gw, gu) = 0$

Therefore $gu = gw$and point of coincidence of f and g in $X$is unique.

Hence by the lemma 4.2.21 f and g unique common fixed point in X

Corollary 3.5 (Aage [13]):Let (X, D) be a *dqb*-complete metric space and $f, g: X \to X$ be self mappings satisfying the inequality $D(fx, fy) \leq \alpha\,[D(fx, gx) + D(fy, gy)]$, for all $x, y \in X$ where $\alpha \in [0,1/2)$ such that $k\frac{\alpha}{1-\alpha} < 1$ and k is coefficient of (X, D). If $f(X) \subseteq g(X)$ and $g(X)$is *dqb*-complete subspace of X, then f and $g$have unique point of coincidence in X. In addition if f and $g$are weakly compatible, then f and $g$have common unique fixed point in X.

Theorem3.6 (Sastry[12]): Let (X, D) be a *dqb*-complete metric space with coefficient $k > 1$. Let $f, T: X \longrightarrow X$ be self-mappings such that T is one-one, $T(X)$ is closed and f is $T\phi$ −contraction then f has unique fixed point.

Corollary 3.7 (Aage[13]): Let (X, D) be a *dqb*-complete metric space with coefficient $k \geq 1$. Let $f, T: X \longrightarrow X$ be self-mappings such that T is continuous, one-one and f is continuous TBanach contraction with $k\alpha \leq 1$. If T is *dqb*-sub-sequentially convergent then f has unique fixed point in X.

Corollary 3.8 (Sastry[12]): Let (X, D) be a *dqb*-complete metric space with coefficient $k \geq 1$. Let

$f, T: X \longrightarrow X$ be self-mappings satisfying T is one-one, $T(X)$ is closed and f is T-Kannan contraction then f has unique fixed point.

Corollary3.9 (Aage[13]):Let (X, D) be a *dqb*-complete metric space with coefficient $k \geq 1$. Let $f, T: X \longrightarrow X$ be self-mappings such that T is continuous, one-one and f is continuous TBanach contraction$T$ −Kannan contraction with $k\alpha \leq 1$. If T is *dqb*-sub-sequentially convergent then f has unique fixed point.

Now we state and prove another main result

Theorem 3.10: Let (X, D) be a *dqb*-complete metric space with coefficient$k \geq 1$. Let $f: X \longrightarrow X$ be self-mapping satisfying $D(fx, fy) \leq \phi\left(max\left\{D(x,y), D(fx,x), D(y,fy), \frac{D(x,fy)+D(fx,y)}{2}\right\}\right) \forall\, x, y \in X$ where $\phi$is altering distance function. Then f has unique fixed point in X.

Proof:

Let $x_0 \in X$ Define $\{x_n\}$ by $x_1 = fx_0, x_2 = fx_1, \ldots\ldots x_n = fx_{n-1}, x_{n+1} = fx_n \ldots\ldots n = 0,1,2\ldots..$

Consider

$D(x_1, x_2) = D(fx_0, fx_1) \leq \phi\left(max\left\{D(x_0,x_1), D(fx_0,x_0), D(x_1,fx_1), \frac{D(x_0,fx_1)+D(fx_0,x_1)}{2}\right\}\right)$

$$= \phi(max\{D(x_0,x_1), D(x_1,x_0), D(x_1,x_2), \frac{D(x_0,x_1)+D(x_1,x_1)}{2}\})$$

$$\leq \phi(max\{D(x_0,x_1), D(x_1,x_0)\}) = \phi\,(\alpha_1\,)$$

where $\alpha_1 = max\{D(x_0,x_1), D(x_1,x_0)\}$

$D(x_2, x_1) = D(fx_1, fx_0)$

$$\leq \phi(max\{D(x_1,x_0), D(fx_1,x_1), D(x_0,fx_0), \frac{D(x_1,fx_0)+D(fx_1,x_0)}{2}\})$$

$$= \phi(max\{D(x_1,x_0), D(x_2,x_1), D(x_0,x_1), \frac{D(x_1,x_1)+D(x_2,x_0)}{2}\})$$

$$\leq \phi(max\{D(x_1,x_0), D(x_0,x_1)\})$$

$= \phi\,(\alpha_1)$ where $\alpha_1 = max\{D(x_0,x_1), D(x_1,x_0)\}$

$D(x_2, x_3) = D(fx_1, fx_2)$

$$\leq \phi(max\{D(x_1,x_2), D(fx_1,x_1), D(x_2,fx_2), \frac{D(x_1,fx_2)+D(fx_1,x_2)}{2}\})$$

$$= \phi(max\{D(x_1,x_2), D(x_2,x_1), D(x_2,x_3)\})$$

$$\leq \phi(max\{D(x_1,x_2), D(x_2,x_1)\}) = \phi\,(\alpha_2)$$

where $\alpha\_2 = max\{D(x_1,x_2), D(x_2,x_1)\}$

similarly $D(x_3,x_2) \leq \phi\,(max\{D(x_2,x_1), D(x_1,x_2)\}) = \phi\,(\alpha_2)$

where $\alpha_2 = max\{D(x_1,x_2), D(x_2,x_1)\}$

Therefore $\alpha_2 \leq \phi(\alpha_1)$

where $\alpha_2 = max\{D(x_1,x_2), D(x_2,x_1)\}$

Therefore $\alpha_3 \leq \phi(\alpha_2) \leq \phi^2(\alpha_1)$

In general $\alpha_n \longrightarrow 0\ as\ n \longrightarrow \infty$

Therefore $\alpha_{n+1} \leq \phi^n(\alpha_1) \longrightarrow\ as\ n \longrightarrow \infty$

Therefore $\alpha_n \longrightarrow 0\ as\ n \longrightarrow \infty$

$$max\{D(x_{n-1},x_n), D(x_n,x_{n-1})\} \longrightarrow 0\ \ as\ n \longrightarrow \infty$$

$$D(x_{n-1},x_n) \longrightarrow 0\ \ as\ n \longrightarrow \infty$$

$$D(x_n,x_{n-1}) \longrightarrow 0\ \ as\ n \longrightarrow \infty$$

Consider for $m, n \in\ N,\ n > m,\ m = n + s$

Now $D(x_{n+1}, x_n) = D(fx_n, fx_{n-1}) \leq \phi(max\{D(x_n,x_{n-1}), D(fx_n,x_n), D(x_{n-1},fx_{n-1}), \frac{D(x_n,fx_{n-1})+D(x_{n-1},fx_n)}{2}\})$

$$= \phi(max\{D(x_n,x_{n-1}), D(x_{n+1},x_n), D(x_{n-1},x_n), \frac{D(x_n,x_n)+D(x_{n-1},x_{n+1})}{2}\})$$

$$= \phi(max\{D(x_n,x_{n-1}), D(x_{n-1},x_n)\})\$\backslash\backslash$$

Now

$D(x_n, x_{n+1}) = D(fx_{n-1}, fx_n) \leq \phi(max\{D(x_{n-1},x_n), D(fx_{n-1},x_{n-1}), D(x_n,fx_n)\frac{D(x_{n-1},fx_n)+D(x_n,fx_{n-1})}{2}\})$

$$= \phi(max\{D(x_{n-1}, x_n), D(x_n, x_{n-1}), D(x_n, x_{n+1})\})$$

$$= \phi(max\{D(x_{n-1}, x_n), D(x_n, x_{n-1})\})$$

$$D(x_{n+2}, x_n) \leq\ k\, D(x_{n+2}, x_{n+1}) + k\, D(x_{n+1}, x_n)$$

$$\leq k\,\alpha_{n+1} + k\,\alpha_n$$

$$\leq k\phi^n(\alpha_1) + k\phi^{n-1}(\alpha_1)$$

$D(x_n, x_{n+2}) \leq k\, D(x_n, x_{n+1}) + k\, D(x_{n+1}, x_{n+2})$

$\leq k\, \alpha_n + k\, \alpha_{n+1}$

$\leq k\phi^{n-1}(\alpha_1) + k\phi^{n}(\alpha_1)$

$D(x_{n+3}, x_n) \leq k\, D(x_{n+2}, x_{n+2}) + k\, D(x_{n+2}, x_n)$

$\leq k\, \alpha_{n+2} + k\,(k\phi^{n}(\alpha_1) + k\phi^{n-1}(\alpha_1))$

$\leq k\phi^{n+1}(\alpha_1) + k^2\phi^{n}(\alpha_1) + k^2\phi^{n-1}(\alpha_1)$

$< k^2\phi^{n+1}(\alpha_1) + k^2\phi^{n}(\alpha_1) + k^2\phi^{n-1}(\alpha_1)$

$= k^2 \frac{\alpha_1}{k^{n+1}} + k^2 \frac{\alpha_1}{k^{n}} + k^2 \frac{\alpha_1}{k^{n-1}}$

$= \frac{\alpha_1}{k^{n-1}} + \frac{\alpha_1}{k^{n-2}} + \frac{\alpha_1}{k^{n-3}}$

$D(x_n, x_{n+3}) \leq k\, D(x_n, x_{n+2}) + k\, D(x_{n+2}, x_{n+3})$

$\leq k\left(k\phi^{n-1}(\alpha_1) + k\phi^{n}(\alpha_1)\right) + k\, \alpha_{n+2}$

$\leq k^2\phi^{n-1}(\alpha_1) + k^2\phi^{n}(\alpha_1) + k\phi^{n+1}(\alpha_1)$

$< k^2\phi^{n-1}(\alpha_1) + k^2\phi^{n}(\alpha_1) + k^2\phi^{n+1}(\alpha_1)$

$= k^2 \frac{\alpha_1}{k^{n-1}} + k^2 \frac{\alpha_1}{k^{n}} + k^2 \frac{\alpha_1}{k^{n+1}}$

$= \frac{\alpha_1}{k^{n-3}} + \frac{\alpha_1}{k^{n-2}} + \frac{\alpha_1}{k^{n-1}}$

Therefore $D(x_{n+s}, x_n) \leq k^{s-1}\phi^{n-1}(\alpha_1) + k^{s-2}\phi^{n}(\alpha_1) + k^{s-3}\phi^{n+1}(\alpha_1) + \ldots + k\phi^{n+s-3}(\alpha_1)$

Therefore $D(x_n, x_{n+s+1}) \leq k^{s}\phi^{n-1}(\alpha_1) + k^{s-1}\phi^{n}(\alpha_1) + k^{s-2}\phi^{n+1}(\alpha_1) + \ldots + k\phi^{n+s-2}(\alpha_1)$

Therefore $D(x_{n+s}, x_n) \leq \alpha_1 \left[\frac{1}{k^{n-s}} + \frac{1}{k^{(n-s)+1}} + \right.$ *1kn−s+2+… +1kn−2+1kn−1→0 as n→∞.*

Therefore $\{y_n\}$ is *dqb*-cauchy sequence in X.

Therefore $\exists\, v \in X \ni x_n \to v$

Now
$D(fx_n, fv) = D(x_{n+1}, fv) \leq$
$\phi(max\{D(x_n, v), D(fx_n, x_n), D(v, fv), \frac{D(x_n, fv) + D(fx_n, v)}{2}\})$

$= \phi(max\{D(x_n, v), D(x_{n+1}, v), D(v, fv)\})$

Letting $n \longrightarrow \infty$

$D(v, fv) \leq \phi(max\{D(v, v), D(v, v), D(v, fv)\})$

$= \phi(max\{D(v, v), D(v, fv)\})$

Similarly
$D(fv, v) \leq \phi(max\{D(v, v), D(v, v), D(fv, v)\})$

Since $x_n \longrightarrow v \Longrightarrow D(v, v) = 0$

Therefore $D(fv, v) = 0$

Therefore $D(v, fv) = 0$

Therefore $fv = v$

v is a fixed point of f

Now we prove that fixed point of f is unique.

Assume that $w \in X$ is another fixed point.

$i.e\ fw = w$

Now
$D(v, w) = D(fv, fw) =$
$\phi(max\{D(v, w), D(fv, v), D(w, fw), \frac{D(v.fw) + D(fv, w)}{2}\})$

$\leq \phi(max\{D(v, w), D(v, v), D(w, w)\})$

By lemma we get $D(v, w) = 0$

Similarly $D(w, v) = 0$

$v = w.$

Corollary 3.11(Aage [13]):Let (X, D) be a *dqb*-complete metric space with coefficient $k \geq 1$. Let $f: X \longrightarrow X$ be self-mapping satisfying $D(fx, fy) \leq \alpha\left(D(x, y), D(fx, x), D(y, fy)\right) \forall\, x, y \in X$ where $\alpha \in [0,1)$ such that $k\alpha \leq 1$. Then f has unique fixed point in X.

Conclusion:

Finally this article can be concluded with the following observations. The purpose of this paper to obtain some new fixed point theorems in dislocated quasi b- metric space was fulfilled. Incidentally we have obtained the results of Sastry [12] and Aage [13] as corollaries.

Conflict of interest

The authors declare that there is no conflict of interest.

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