A Note on illustrations of Elasticity

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Abstract*:* The concept of elasticity in economics is taught mostly in secondary education. Students generally mix up slope and elasticity even though many textbooks explain the difference between them. This note is to substantiate explanations by Round and McIver (2006) in view of exploring possible reasons how students might have confused elasticity and slope from initial knowledge of elasticity. This note indicates some misunderstandings that arise from elasticity illustrations, and also provides possible and alternative ways for understanding elasticity. As indicated by Round and McIver (2006), a precise use of terms in textbooks is necessary to understand the difference between the elasticity and slope of a demand curve in third degree price discrimination, given the assumptions in which the theory has been established in relation to elasticity.

Key words: Elasticity, demand, supply, slope, monopoly, price discrimination

Elasticity is an economic concept that is explained in many economics textbooks. There is a slight misconception in distinguishing elasticity of a demand from slope of the demand curve even though textbooks clearly state elasticity is different from slope. Many confuse these terms with geometrical shapes though they know that slope is a determinant of elasticity. This note provides a guideline to understand the difference between elasticity and slope, and some other related aspects in elasticity.

The elasticity of demand is measured as a ratio of percentage change in quantity over percentage change in price of a product. It is well known that the formula of elasticity consists of slope of a straight line. But, understanding slope and elasticity are mostly mixed up when graphical interpretations are given. Round and McIver (2006) identify how students confuse elasticity with slope of a demand curve, when they discuss particularly third-degree price discrimination in a classroom discussion. This endorses the existence of confusion in differentiating elasticity from slope. The purpose of this note is to substantiate their discussion and explore how the misunderstanding between slope and elasticity is misperceived from initiation of knowing elasticity. Though many understand slope and elasticity are different throughout the textbooks, they sometimes fail to accept a fact that any sloped-simple-demand curve consists of five (5) different types of elasticity measures along the line.

The misunderstanding of the difference between slope and elasticity mainly arises when they are illustrated in graphs. In illustrations, it is important which part of the (5 types of) elasticity is mainly focused on a line and these explanations should not be mixed up with slope of the line. If one uses an illustration of elasticity, he/she should explain why such pattern of demand curve becomes specific to that particular type of elasticity, provided that a sloped-simple demand line has all types of elasticity. In this context, this note carries some important explanations in view of providing more insights on elasticity.

The following sections are organised as the basic elasticity formula, misunderstanding in elasticity, remedial steps, and conclusion.

The basic Elasticity Formula

Elasticity (E) is measured as:

$$E = \left(\frac{\textit{Percentage Change in Quantity}}{\textit{Perntage Change in Price}}\right) = \left(\frac{\Delta Q}{Q}\right) \div \left(\frac{\Delta P}{P}\right) = \left(\frac{\Delta Q}{Q}\right) \times \left(\frac{P}{\Delta P}\right)$$

The above measure can be extracted as:

$$E = \left(\frac{\Delta Q}{\Delta P}\right) \times \left(\frac{P}{Q}\right)$$

i.e., slope of a straight line is multiplied by the ratio of price to quantity at a point on the line.

Laudadio (1968) has devised an equivalent alternative way to measure a coefficient of elasticity for a price on a demand curve. Though it is a simple approach, Laudadio (1968) does not directly explain how the same approach can be used to determine the elasticity of supply. Anyhow, Laudadio's (1968) suggestion shows that minimum two measures are sufficient to determine a coefficient of elasticity: (1) the price at which the coefficient of elasticity is required, and (2) the value of Y-intercept.

The price elasticity is related to price – quantity relationship in economics. But, it is notable that the determination of elasticity sometimes goes beyond such a relationship because of different rearranging options of elasticity formula. Therefore, it is important to focus on the primary equation shown above to keep track on the price – quantity relationship from an economic point of view.

Misunderstanding in Elasticity

The definition of the elasticity of demand, for example, is generally expressed in two ways.

(a) Based on the formula, the elasticity is defined as a ratio of percentage change in quantity over percentage change in price; and

(b) Based on the causal relationship between quantity and price of a product, elasticity measures the extent to which a price change causes a change in quantity demanded.

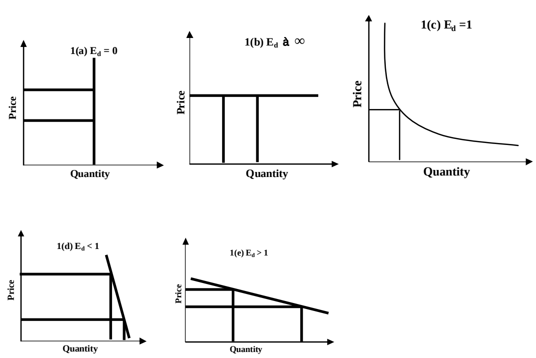
These definitions imply that elasticity needs to be defined with a price change and its causal effect on quantity demanded. In the above two definitions, the (a) just guides to measure elasticity and (b) explores the causal effect of the price change on quantity. It is notable that both definitions appear to be not capturing the explanation for point elasticity.

Students relate slope of a simple demand curve with elasticity when illustrating elasticity with sloped demand curves in particular. Most of the text books highlight that the elasticity and slope cannot be confused with each other, (e.g., Samuelson and Nordhaus, 2005; Gans *et al*, 2005).

This strong suggestion implies an existing possible misunderstanding of the elasticity of demand with its component slope. Even though textbooks have clearly defined and reasoned the difference between elasticity and slope, the illustrations in many of them are not compatible with the view why such confusion arises. Further, they do not explore the commonality of graphical illustrations for both the elasticity of demand and supply. For an easy understanding, the different illustrations in elasticity of supply and demand should be brought down into a general format.

In elasticity illustrations, five (5) different shapes of demand curves are initially illustrated in many textbooks. They are given in Figure 1.

Figure 1: Elasticity of Demand in Formal Illustration



Though the elasticity is explained as a ratio of percentage changes between price and quantity in demand, the different sloped demand curves with elasticity identifications in Figure 1 dilute distinguishing elasticity measures from slopes of the demand curves. In the illustrations in Figure 1, the demand curves 1(d) and 1(e) are in particular subject to constant slopes. As a simple sloped demand curve consists of all 5 different elasticity measures and an elasticity measure takes place on a straight line basis, the shapes of the demand curves in 1(d) and 1(e) are mixed up with the slope of the demand curve. This seems like a denial of existing 5 different elasticity measures on a sloped demand curve.

The elasticity measure highlights the ratio between percentages of change in quantity and change in price. While an economic principle in elasticity underlines the responsive relationship of price as indicated in 1(d) and 1(e), illustration 1(c) is not compatible with 1(d) and 1(e) because 1(c) indicates for unit point elasticity. If price – quantity relationship is highlighted with two points in 1(c), the unit elasticity may not be available at a point of price. For a responding relationship of price with quantity, the unit elasticity (E_d = 1) can possibly be shown with a simple straight line demand curve by pointing out the middle point of the demand line between X and Y axes.

In another way, if the illustrations in Figure 1 refer to point elasticity as shown in 1(c), it is not necessary to show a demand curve with slope in illustrations because the measures of elasticity E_d<1 and E_d>1 are identifiable on any simple sloped straight demand curve. The approach shown in 1(d) and 1(e) may not be compatible with 1(c) to explore price – quantity relationship in point elasticity. Hence, this may possibly cause for confusion in distinguishing elasticity from slope in illustrations.

It is possible to argue that low slope (flattered) refers to high elasticity and vice versa. In elasticity of demand, high price associates with high elasticity and low price does with low elasticity. It is notable whether the demand line is flattered or steeped (formally referred as low elasticity) and that line consists of both low prices and high prices.

This implies that any sloped demand curve contains all 5 types of elasticity on it. Hence, the argument low slope refers to high elasticity and vice versa is not true all the time. If, for example, one assesses elasticity on a medium sloped (say at tan 45^o as not steeped or flattered) demand line, he/she cannot conclude that such a medium slope line can represent a unit elasticity measure. The same argument is inappropriate in case of a supply curve. Note that a supply curve with negative intercept on Y-axis and a flattered slope can be an example of low elasticity ($E_S < 1$) because of the negative intercept. In this context, one can observe that low elasticity of supply is resulted in because of the negative intercept, not by the slope of the supply line.

In textbooks, no common illustrative approach is available to identify elastic and inelastic areas of demand and supply curves. Generally in this context, students perceive elasticity of demand curves with slope and elasticity of supply curves with the Y-intercepts. In elasticity of supply (E_S), three (3) illustrations ($E_S < 1$, $E_S = 1$, and $E_S > 1$) of those five (5) types (except 0 and ∞ elasticity) are completely different from the approach adapted to explain the elasticity on a demand curve. For simple recognitions of elasticity of a supply curve, one can asses at which point the supply curve intercepts Y-axis (the price measure for zero quantity). When the Y-intercept of price is positive (> 0), the simple straight line supply curve is elastic (E_S>1). Similarly, when Y-intercept = 0, the supply curve is of a unit elastic ($E_S = 1$) and if Y intercept is negative (< 0), the supply curve is inelastic (E_S<1). While a straight demand curve with a definite slope consists of all (5) elasticity types on it, a simple supply curve cannot have this. This again dilutes understanding illustrative principles in elasticity. But, the illustrations of elasticity for supply curves are good examples to explore the slope as an element of elasticity and the difference between slope and elasticity.

As Laudadio's (1968) approach is simple to determine elasticity of demand at a price, it indirectly explores a possibility of determining the elasticity of supply. Hence, it is wise to find ways for how that approach can be used in an illustration (in geometry) to determine elasticity of supply to establish a consistency with determining elasticity of demand. A graphical illustration may also provide a clear idea to understand Laudodio (1968) approach (refer to Appendix 1).

Overlapping of different measures in elasticity, such as point elasticity and arc (average price) elasticity in particular can also be confused to a certain extent. For instance, assuming a straight line demand curve, consider that a price changes from P_1 to P_2. Based on this, point elasticity at price P_1 and P_2, arc elasticity of P_1 and P_2, and price change elasticity of P_1 towards P_2 can be determined. In this context, it is possible to argue that the point elasticity can be referred to as the elasticity measure at the price of P_1, P_2, or the average of (P_1+P_2). It is clear that the price change elasticity and arc elasticity can also be termed as point elasticity. Hence, it is necessary to explore the possibility in which the differences between elasticity measures are identifiable.

Round and McIver (2006) explore students' misunderstanding of elasticity with slope and emphasise that textbooks should provide précised explanations for why two prices are associated with two different groups in third degree price discrimination. It is noted that using the terms 'elastic' and 'inelastic', when explaining price discrimination, might have caused for further confusion between elasticity and slope, (e.g., Samuelson and Nordhaus, 2005).

Based on the range of elasticity, only three major types of goods are identifiable: (a) elastic goods, (b) unit elastic goods, and (c) inelastic goods. In some text books, the terms ‘elastic’ and ‘inelastic’ are used to identify two different market groups in third degree price discrimination. In simple terminology, textbooks refer to the term ‘elastic’ for the goods of elasticity more than one and ‘inelastic’ for that of less than one. When the same terms are used to distinguish two different markets in monopoly, it simply accepts that one of two groups (markets) has less than one ($E_d<1$ = inelastic) and the other has more than one ($E_d>1$ = elastic) elasticity coefficient. In this context, many textbooks do not focus on the fact that a monopoly market conditionally operates on the situation where its products has elasticity more than one on the demand curve with positive marginal revenue (MR), i.e., on the ‘elastic’ area (($E_d>1$) of the demand curve, (refer to Appendix 2). Using the terms ‘elastic’ and ‘inelastic’ is inappropriate for two groups in price discrimination because the monopoly market aims to maximise profit where a firm operates at MR>0 with $E_d>1$ (elastic) for both groups. Hence, explaining the firm’s operation under the conditions of negative marginal revenue (MR<0) and inelastic product ($E_d<1$) is not theoretically acceptable.

Remedial Steps

Elasticity and Linear Relationship between Price and Quantity

It is noted that the definition of elasticity is generally based on (a) the equation of elasticity, and (b) the price change impact on quantity. Both seem as not incorporating the explanation of point elasticity. If one believes that elasticity measure is a causal effect of price on quantity, he/she may question how a point elasticity measure contains a ratio of change in quantity to change in price though no change in price takes place. Many textbooks have illustrated how point elasticity contains such a ratio of change in quantity to change in price. It is notable that the elasticity measure is based on a straight line, and its slope ($b = \Delta P/\Delta Q$) and a point (Q, P) on the line determine the coefficient of elasticity at a particular price P (refer to Appendix 3 for detail).

Overall, a definition of elasticity should be highlighted with a linear relationship of independent (price or income) and dependent variables (quantity) to determine the elasticity of a point on a curve. No matter what type of a curve those variables form, any elasticity coefficient is determined by the linear relationship of variables that give constant value of slope at any point on the line. The changing value is the ratio of price to quantity that needs to satisfy the straight line on which elasticity calculation depends. But, it is not necessary that the point needs to satisfy the price – quantity relationship curve. Hence, this note addresses that definition of elasticity should incorporate linear relationship of variables. Thus, elasticity for a price can be defined as a point measure on a straight line relationship between price (income) and quantity that indicates a ratio of expected responsive percentage change in quantity to expected percentage price change along the straight line.

Illustration of Elasticity

While illustrating different types of elasticity, it is necessary to accept the fact that any sloped simple demand curve consists of all 5 types of elasticity. If one cannot realise this true nature of elasticity, there is a misunderstanding.

Figure 2 for a simple demand curve shows how this can be understood. The illustration shows, irrespective of slope of a demand curve, which part of the curve really highlights a required range of elasticity, compared to other elasticity measures. In formal illustration, such comparison is lost or cannot be realised and this may cause confusing elasticity with slope of a demand curve. Note that the rectangular hyperbolic demand curve can only represent unit elasticity at every point on it and that cannot be used to indicate price change elasticity.

The formal illustrations reflect which part of a demand curve represents a particular range of elasticity, but they do not indicate unnecessary ranges of elasticity. These illustrations therefore recognise different types of sloped demand curves. Generally from an elasticity point of view, demand curves are categorised as curvy and simple straight line demand curves.

The straight line demand curves can also be further categorised as vertical (for zero elasticity), horizontal (for perfect elasticity), and sloped (to represent the elasticity range as for $0 \leq E_d \leq \infty$) lines. Improper identification of different demand curves for different elasticity might have let to the perception that a sloped demand line consists of a particular elasticity throughout the demand curve between X and Y axes. This perception is a result of the sloped based identification of different types of elasticity in demand. Note that elasticity needs to be categorised, not the different sloped demand curves for different elasticity.

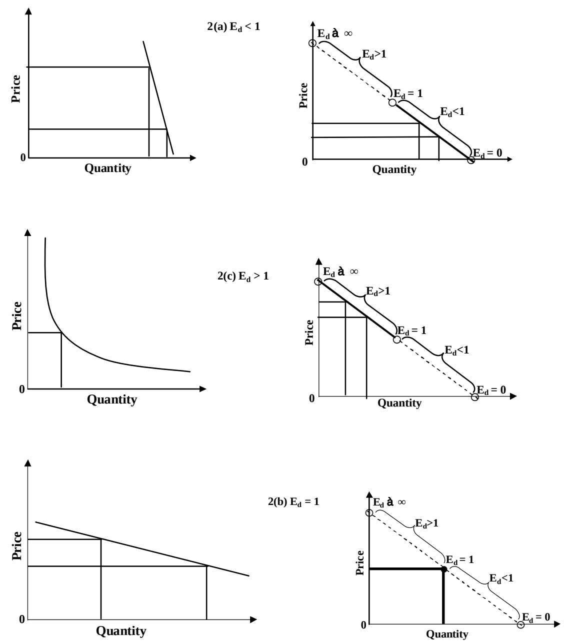
For elasticity of supply, the $E_s<1$, $E_s=1$, and $E_s>1$ are identically differentiated from slope because in simple linear illustration, these types of elasticity are recognised with Y-intercept of supply curve. In this context, it is important to know how the approach used in supply, to differentiate slope and elasticity, can be compromised with demand curves.

Laudadio's (1968) approach is also applicable to find elasticity of supply. For a general supply equation $P_s = h + kQ_s$, in this approach, the elasticity at a point of price and quantity is determined as $E_s = P_s/(P_s - h)$. For elasticity of demand (assume $P_d = a - bQ_d$) at a price is $E_d = P_d/(a\text{-}P_d)$. Both elasticity of demand and supply are based on a particular price at which elasticity is required and the Y-intercept, 'h' in supply and 'a' in demand equations. This is a common algebraic approach to determine elasticity, irrespective of slope of a curve.

Appendix 1 explains how demand elasticity of a price at $T(q_1, p_1)$ can be determined when it is located between two points (S and R) on X and Y axes respectively. This application is useful to determine point elasticity on a curvy demand line. It is possible to question how the same approach is useful to determine point elasticity on a curvy supply curve. This is identical as similar to determining point elasticity on a curvy demand line.

Figure 2: Alternative ways to show elasticity of $E_d<1$, $E_d=1$, and $E_d>1$

Formal Illustration Alternative Illustration to



Consider a curvy supply function Q_s = f(price) to determine elasticity at a point U(q_2, p_2). Because of a negative relationship between quantity demanded and price, it is possible to have a tangent to the curvy demand line at the point where elasticity needs to be determined (as shown in A1.2 of Appendix 1). As the supply curve has a positive relationship between quantity and price, it is essential to have tangent line (BC) to Q_s = f(price) through the point U(q_2, p_2) at which the elasticity is to be determined, (refer to the Figure 3). To implement a similar approach for elasticity determination, irrespective of sign (+ or –), the same sloped demand type straight line to the tangent of supply curve needs to be identified. For this, following steps can be followed (refer to Figure 3).

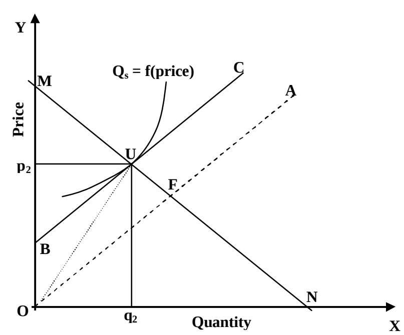
a) Draw a parallel line OA to the tangent line BC, i.e., OA||BC, through the origin.

b) Draw a line MN through point U on condition that MF = FN, (where the points U and F are on MN, and F is the middle point of MN).

As the absolute slope of MN is equal to the tangent slope of supply curve (BC), the price elasticity at point U on the supply curve can be measured as (NU/MU) as shown in Appendix 1.

To conciliate both illustrative approaches in demand and supply in determining elasticity, an intercept (point F) of a supply type (namely OA) and a demand type (namely MN) straight lines become bases to determine elasticity at a point of price. With a transformation of elasticity formula (refer to Appendix 4) , it is possible to show that price elasticity at point U(q_2, p_2) is the ratio of slopes of lines OU and BU as (OU_{slope} /BU_{slope}). This method is also common for both in determining the elasticity of demand and supply.

Figure 3: Compromising elasticity of supply with elasticity of demand



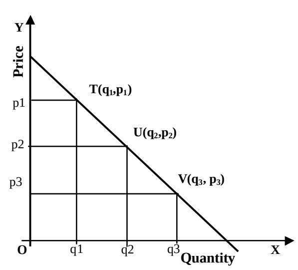
Identifying Different Elasticity Coefficients

An elasticity measure takes place at a point only, irrespective of how that particular point is selected. As this is a general theoretical aspect, one may confuse point elasticity with price change elasticity and arc (average) price elasticity. This note explores the situation in which a price change elasticity and arc elasticity can be termed as point elasticity. This depends on how we consider a demand curve, for example, in determining elasticity at a point of price. Generally, only two types of demand curves are considered for a product in illustrations: (a) Linear relationship of price with quantity – this is a straight line approach for a demand curve; (b) Curvy linear relationship – this is not a straight line approach.

When a demand curve is presumed as a simple straight line, any elasticity measure along the line is point elasticity at any particular price. Consider a straight line demand curve (RS) and three points on it as $T(q_1, p_1)$, $U(q_2, p_2)$, and $V(q_3, p_3)$ where point U is on TV and represents average (midpoint) price and quantity of points T and V (refer to Figure 4).

The elasticity at points T, U, and V are the point elasticity at respective prices. When price changes from p_1 to p_3, the price change elasticity and the point elasticity are measurable at the price of point T and both are same. If the (arc) elasticity for average price $[p_2=(p_1+p_3)/2]$ is determined, it can be termed as the point elasticity of p_2 that is measured for the average of new (p_3) and initial (p_1) prices.

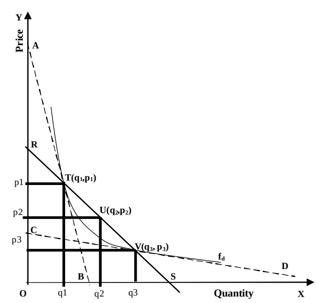
Figure 4: Point elasticity at different prices



Notably, all three points are on a straight line demand curve and any elasticity measure that takes place along the line at a respective price is point elasticity. This is entirely different from determining different elasticity measures when a demand line is a curvy linear.

Consider a curvy linear demand curve (f_d) in application and only points T and V are on it (refer to Figure 5). The price p_1 (at T) on f_d changes to p_3 (at V) and the average of those prices p_1 and p_3 is p_2 (at U). The line TUV represents the price change and meets X and Y axes at S and R respectively.

Figure 5: Different Measures of Elasticity



Because demand is curvy linear, the price change elasticity is measured based on the slope of the line RS. But, the point elasticity at p_1 (in T) is estimated based on the slope of line AB which is a tangent of f_d at point T. As the slopes of lines AB and RS are not the same, the price changed elasticity at point T with the slope of RS is not the same as the point elasticity at T with the slope of AB. Similarly, the point elasticity at point V (price p_3) is not the same if the price change elasticity is determined at V. The arc elasticity also deviates from point elasticity if demand is represented with the curvy line in application. A price change (p_1 to p_3) does not keep the average price (p_2) on the demand curve (f_d). The arc elasticity is also determined with the slope of RS through which price changing positions take place. But, arc elasticity is different from a view of point elasticity at price p_2 on f_d.

Price discrimination: Does it relate to elastic and inelastic for two groups (markets)?

In a monopoly, no firm entertains negative marginal revenue (MR<0) in a market. This situation is applicable to third degree price discrimination too (refer to Appendix 2). Hence, the price discrimination takes place where MR>0 and this becomes possible when $E_d>1$ only. This implies that the determination of two prices in discrimination takes place only on the area of $E_d>1$, i.e., in the 'elastic' part of the demand curves for both groups (markets). The only difference is that one group (market) has relatively more elasticity than the other. Round and McIver (2006) have strongly emphasised this consistently with the condition that a monopoly operates when $E_d>1$ where MR>0 and spell out the price discrimination as a result of the differing values of elasticity in two groups (market). Hence, this note suggests to use the terms 'more' and/or 'less' in comparison of elasticity for two discriminated prices particularly in monopolistic competition, not the terms 'elastic' (formally means $E_d>1$) and 'inelastic' (formally $E_d<1$).

Conclusion

This note focuses on misunderstanding of elasticity with its slope component. Though it is understood that slope is an element of determining elasticity at a point of price and quantity, elasticity of demand and elasticity of supply are not illustrated in a common illustrative perspective. Particularly, the formal illustrations on both demand and supply in particular dilute clarity in compromising approaches in them. To make a common approach, it is necessary to accept that an elasticity measure takes place along a demand-shaped straight line, which consists of all major five (5) elasticity types on that line. The formal illustrations do not deny this fact. But, they do not provide a common graphical illustration for elasticity of demand and elasticity of supply in comparison. This note emphasises that different illustrations in elasticity of demand and supply should be brought into a general format for an easy understanding of elasticity and to distinguish elasticity from slope of a line

While explanations are extended to show a common illustrative approach applicable to show both elasticity of demand and supply, the overlapping explanations of point elasticity with price-change and arc elasticity are additionally explained in view of showing the difference in them. This note considers a distinguishing terminology of price (or income)-change elasticity from point elasticity and indicates that definition of elasticity should be acceptable to accommodate explanation of point elasticity.

Finally, this note has focused on the terms 'elastic' and 'inelastic' used in textbooks in third degree price discrimination and suggests to avoid those terms because the formal explanations for 'elastic' ($E_d>1$) and 'inelastic' ($E_d<1$) have no valid theoretical background in third degree price discrimination, since a monopoly market is always found with the situation MR>0 where $E_d>1$ and is applicable to third degree price discrimination too. In theory, a monopolistic firm operating with $E_d<1$ is not critically accepted.

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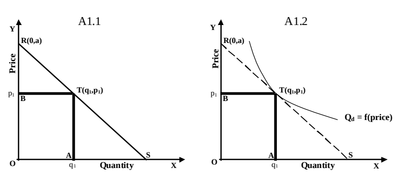
Appendix 1

The appendix illustrates how formal equation of elasticity and Laudodio (1968) are similar. Consider a simple demand equation P = a – bQ for a product and a point $T(q_1, p_1)$ on it. The points R and S are intercepts of axes Y (price) and X (quantity) respectively; and the quantity and price of point T are at A (p1) and B (q1) respectively, (refer to the Fig. A1).



As the point $T(q_1, p_1)$ is on the demand curve P = a – bQ,

$p_1 = a - bq_1$ à $bq_1 = (a - p_1) = BR$; and $p_1 = BO$

As BT||OS, the point T geometrically divides line RS at the ratio (ST : RT) = (BO/BR)

$$= (ST/RT) = (BO/BR) = p_1/(a - p_1) = \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P}\right)$$

This approach is same for determining a point elasticity on a curvy demand curve [Q_d = f(price)] when a tangent of the f(price) goes through the point $T(q_1, p_1)$ as shown in A1.2.

This result is the same as indicated by Laudodio (1968). Hence, the elasticity is a measure that varies from zero (0) to infinity (∞) in accordance with the movement of point T along the straight (demand) line RS. As points S and R are stable, the price (in point T) movement along the line determines the level of elasticity as (ST/RT). Similarly, it is possible to show that an elasticity coefficient on a supply curve (E_S) can also be determined as $E_S = P/(P - h)$, as given general equation of a supply curve $P_S = h + kQ_S$.

Appendix 2

In monopoly or monopolistic competition, the price (P = average revenue AR) and quantity (Q) of a product are interdependent on each other in determining the total revenue (TR).

Therefore, ***TR = P * Q***

For a monopolistic firm, marginal revenue (MR) needs to be positive for its realistic operation (for avoiding loss). Differentiating TR with respect to quantity (Q) results in as:

$$\frac{d(TR)}{dQ} = MR = P\left(\frac{dQ}{dQ}\right) + Q\left(\frac{dP}{dQ}\right) = P + Q\left(\frac{dP}{dQ}\right)$$

…………. (eq1)

But $E_d = -\left(\frac{dQ}{dP} \times \frac{P}{Q}\right)$ …………. (eq2)

Note that the negative value of E_d is transformed into its absolute term as multiplied by a negative.

From the (eq2), $\frac{dP}{dQ} = \frac{-P}{Q\,E_d}$ and substituting this in (eq1) results in:

$$MR = P + Q\left(\frac{-P}{Q\,E_d}\right) = \left(P - \frac{P}{E_d}\right) = P\left(1 - \frac{1}{E_d}\right)$$

$$MR = P\left(1 - \frac{1}{E_d}\right)$$ …………. (eq3)

The realistic operation of a firm is confirmed with MR>0 because the equilibrium of a firm is confirmed with MR= marginal cost MC. As a firm in monopoly does not reach its equilibrium, the firm always keeps its MR>0. The (eq3) confirms the absolute value of elasticity needs to be greater than 1 (E_d>1) if the firm is away from loss. For any E_d<1, the MR is negative. The firm does not entertain a group that provide negative MR.

The condition in (eq3) cannot be violated in any circumstances and has been accepted in monopolistic competition market, thus applying even for third degree price discrimination.

Appendix 3

Consider demand equation **P = a – bQ**, where **b = (ΔP/ΔQ)**. Appendix 1 confirms that the elasticity of point T(q_1, p_1) is measured based on its location on the line RS, where R and S are the extreme points of the straight line RS on Y and X axes respectively. So that, $\mathbf{E_D = (ST/RT) = p_1/(a - p_1)}$, where $\mathbf{a = (p_1 - bq_1)}$.

Substituting $\mathbf{a = (p_1 - bq_1)}$ in $\mathbf{(ST/RT) = p_1/(a - p_1)}$,

$$\left(\frac{ST}{RT}\right) = \left(\frac{p_1}{(p_1 - bq_1 - p_1)}\right) = \left|\frac{p_1}{bq_1}\right|$$

But, b = (ΔP/ΔQ) and therefore,

$$\left(\frac{ST}{RT}\right) = \left|\frac{p_1}{bq_1}\right| = \left|\frac{1}{b} \times \frac{p_1}{q_1}\right| = \left(\frac{\Delta q}{\Delta P} \times \frac{p_1}{q_1}\right)$$

In general formulation,

$$E_D = \left(\frac{1}{b} \times \frac{P}{Q}\right) = \left(\frac{\Delta Q}{\Delta P} \times \frac{P}{Q}\right) = \left(\frac{\Delta Q}{Q} \times \frac{P}{\Delta P}\right) = \left(\frac{\Delta Q}{Q} \times \frac{P}{\Delta P}\right) = \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P}\right)$$

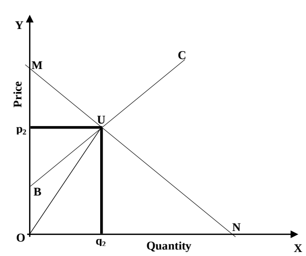
$$E_D = \left(\frac{\text{Percentage Change in Quantity}}{\text{Percentage Change in Price}}\right)$$

This is the formal equation for coefficient of elasticity that confirms how change in quantity and change in price are inclusive in point elasticity on a line on which elasticity considers linear relationship of price and quantity.

Appendix 4

The elasticity formula is: $E_D = \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P}\right) = \left(\frac{P}{Q} \times \frac{\Delta Q}{\Delta P}\right) = \left(\frac{P}{Q} \div \frac{\Delta P}{\Delta Q}\right)$

A3.1



From A3.1, assume MN and BC are demand and supply curves respectively, point U(p_2, q_2) is on them, and slope of any one of them is (Δp/Δq).

Referring to the above formula, the demand elasticity at point U is as:

$$E_D = \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P}\right) = \left(\frac{P}{Q} \times \frac{\Delta Q}{\Delta P}\right) = \left(\frac{P}{Q} \div \frac{\Delta P}{\Delta Q}\right) = \left(\frac{p_2}{q_2} \div \frac{\Delta p}{\Delta q}\right)$$

As O(0, 0) and U(p_2, q_2), the slope of OU is simply (p_2/q_2). And also the slope of demand (or supply) curve at point U is ($\Delta p/\Delta q$). Therefore,

$$E_{D(or\ S)} = \left(\frac{\Delta Q}{Q} \div \frac{\Delta P}{P}\right) = \left(\frac{P}{Q} \times \frac{\Delta Q}{\Delta P}\right) = \left(\frac{P}{Q} \div \frac{\Delta P}{\Delta Q}\right) = \left(\frac{p_2}{q_2} \div \frac{\Delta p}{\Delta q}\right) = \frac{Slope\ of\ OU}{Slope\ of\ MN\ (or\ BC)}$$

This implies that there is consistent in identifying elasticity of demand and supply at a point of price.