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Linear Programming with Post-Optimality Analyses – A Case Study

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Wilson Manufacturing produces both baseballs and softballs, which it wholesales to vendors around the country. Its facilities permit the manufacture of a maximum of 500 dozen baseballs and a maximum of 500 dozen softballs each day. The cowhide covers for each ball are cut from the same processed cowhide sheets. Each dozen baseballs require five square feet of cowhide (including waste), whereas, one dozen softballs require six square feet of cowhide (including waste). Wilson has 3600 square feet of cowhide sheets available each day.

Production of baseballs and softballs includes making the inside core, cutting and sewing the cover, and packaging. It takes about one minute to manufacture a dozen baseballs and two minutes to manufacture dozen softballs. A total of 960 minutes is available for production daily.

1. Formulate a set of linear constraints that characterize the production process at Wilson Manufacturing.

Decision Variables

X_1= number of dozen baseballs produced daily

X_2= number of dozen softballs produced daily

Constraints

In addition to non-negativity constraints (i.e., the implied constraints) for the decision variables, there are three functional constraints.

1. The use of cowhide.
2. The daily limit for production time.
3. The maximum production limit of total units.

Cowhide

The total amount of cowhide used daily cannot exceed the amount of cowhide available daily $5X_1 + 6X_2 \leq 3600$

Production Time

The amount of production minutes used daily cannot exceed the total number of production minutes available daily $X_1 + 2X_2 \leq 960$

Production Limit

The total number of dozen units produced daily cannot exceed the marketing limits

$X_1 \leq 500$

$X_2 \leq 500$

Non-negativity of Decision Variables

Negative Production of baseballs and softballs is impossible. Thus, $X_1, X_2 \geq 0$

The Mathematical Model

Max $7X_1 + 10X_2$ (Objective Function)

Subject to:

$5X_1 + 6X_2 \leq 3600$ (Cowhide)

$X_1 + 2X_2 \leq 960$ (Production time)

$X_1 \leq 500$ (Production limit of baseballs)

$X_2 \leq 500$ (Production limit of softballs)

$X_1, X_2 \geq 0$ (Non-negativity)

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Detailed Step By Step Description of the Problem and

The Graphical Solution Algorithm

Problem Description: The first step in understanding Wilson manufacturing production *choices* is to have a firm understanding of *present limitations*. After an understanding of the limitations is set, one must understand how these limitations affect what one is trying to optimize. A row and column chart works nicely; in this case an understanding of the material and production time, necessary for producing baseballs and softballs. After the limitations are understood and the material and production time of each product is understood, one can then write a linear equation that embodies these limitations.

A variable such as X_1 will represent a baseball production; variable X_2 will represent softball production. By looking at the chart containing the parameters of the problem, one can determine that five square feet of cowhide is needed per dozen of baseballs. Therefore 5 X_1 represents the cowhide needed per dozen of baseballs produced. The same process can be applied to softballs by stating 6 X_2, representing 6 square feet needed per dozen softballs produced. The cowhide available is 360 square feet therefore total baseball and softball production must be less than and or equal to 360 square feet, the linear inequality $5\ X_1 + 6\ X_2 \leq 360$ places all of these conditions into a simple mathematical problem.

Next we turn our attention to time (labor) constraints. By looking at the chart containing the parameters of the problem, we see that it takes 1 minute of production time per dozen baseballs and 2 minutes of production time per dozen softballs. The total time available per day is 960 minutes; therefore total production time of softballs and baseballs must be less than or equal to 960. This will read as a linear equation, $1\ X_1 + 2\ X_2 \leq 960$.

The next constraints to be considered are the ability of the machinery to produce baseballs and softballs. The baseball *machine has a limit* of producing 500 baseballs per day. The softball machine also has a limit of producing 500 softballs per day. The linear equation that embodies these constraints is X_1 (baseballs) must be less than or equal to 500. X_2 (softballs) must be less than or equal to 500. The final constraint is the non- negative constraint, which is critical in understanding production. One cannot make negative product therefore X_1 and X2 must be greater than 0.

After a keen understanding of the constraints and how each constraint applies to the manufacturing problem, ones attention must be turned toward *the goal*, in this case it is a *maximization net profit*. We want to maximize the profit from producing baseballs and softballs. We can see that the net profit on baseballs is 7 dollars per dozen and the net profit on softballs is 10 dollars per dozen. We attach these profit numbers to the

variables that represent baseballs and softballs. This is seen in the literary equation $7\ X_1 + 10\ X_2$ when we add the word maximize in front of that linear equation we have a program that can operate inside of a software application.

Next we turn our attention to making a graph (i.e., the feasible region) of the production problem. The horizontal axis represents baseballs and the vertical axis represents softballs. A vertical line should be drawn that bisects the 500 number mark on the X- axis; this line represents the maximum number of baseballs that can be produced. The horizontal line should be drawn that bisects the 500 mark on the Y-axis this represents the maximum number of softballs that can be produced. Next we graph the time constraint which is determined by drawing a point on the X-axis that represent the case that all the time were spent just making baseballs. Total time available is 960 so the point is at point 960. The Y-axis, which represents the time necessary to make softballs, the point would be at 480. When a

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line is then drawn across the graph plain, this line will represent the time constraint. Finally we have to draw the cowhide restraint. This is determined by finding the point in the X-axis that represents the case if all the cowhide were used on baseballs. This point is 720. Now we draw a line on the Y-axis that represents, if all of the cowhide were used on softballs. This number is 600 we connect those two dots with a line to the graph plain.

Because the cowhide restraint must be less than 3600, all the area to the right of the cowhide constraint line will not be included in the solution. Because the time constraint is equal to or less than 960 all the area above the time constraint line will not be included in the solution. Because the limit of 500 placed on baseballs all the area to the right of the baseball constraint line will not be included in the solution. Because the constraint of 500 is placed on softballs, all the area above the softball constraint line is excluded from the solution. When all of these lines are placed on a graph and the areas that are excluded are removed, including the negative areas, the area that is left is called the feasible region. Any *corner point* of the feasible region can be accomplished under present constraints. In order to maximize profits, a point where two of the line constraints bisect must be chosen. There are four such points in our problems. After subtracting linear equation from linear equation the best point of manufacturing is where the cowhide constraint and the time constraint bisect, this point would be producing 360 baseballs and 300 softballs.

As we know, a Formulation of the Wilson Manufacturing problem is: X_1 = the number of dozen baseballs manufactured daily X_2 = the number of dozen softballs manufactured daily.

Max Objective Function $7 X_1+10 X_2$

Subject to:

$C_1 = X_1 \leq 500$ (constraint of production)

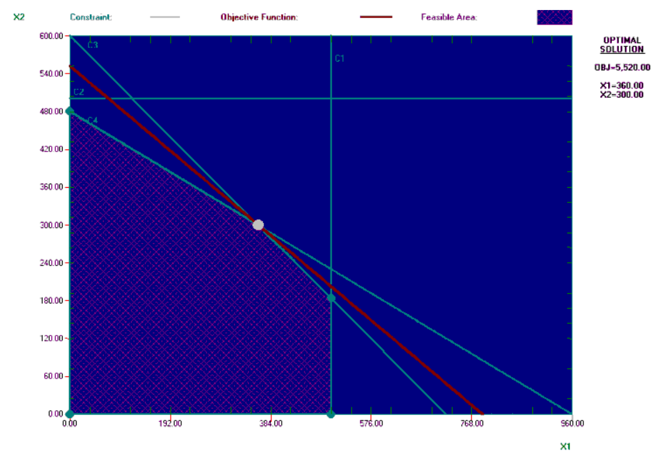
$C_2 = X_2 \leq 500$ (constraint of production)

$C_3 = 5 X_1 + 6 X_2 \leq 3600$ (this is the constraint of the material)

$C_4 = X_1 + 2 X_2 \leq 960$ (the constraint of the production time available)

$C_5 = Xj \geq 0; j = 1, 2$ (non negativity)

a. **Graph the feasible region for this problem (**Hand computation is submitted separately**).** C_4 relegates the solution to quadrant I. X_1 and X_2 must be positive numbers (or 0).



Wilson is considering manufacturing 300 dozen baseballs and 300 dozen softballs. Applying the constraints:

C_1; $X_1 \leq 500$; 300 < 500 Constraint is satisfied.

C_2: $X_2 \leq 500$; 300 < 500 Constraint is satisfied.

C_3; $5 X_1 + 6 X_2 \leq 3600$;

5(300) + 6(300) =3300<3600 Constraint is satisfied.

C_4; $X_1 + 2 X_2 \leq 960$;

300 +2(300) = 900 < 960 Constraint is satisfied.

Since the constraints are satisfied, the solution of 300 dozen each is an interior point located within the feasible area.

Wilson considers manufacturing 350 dozen baseballs and 350 dozen softballs. Again, applying the constraints:

C_1; $X_1 \leq 500$; $350 < 500$ Constraint is satisfied.

C_2; $X_2 \leq 500$; $350 < 500$ Constraint is satisfied.

C_3; $5 X_1 + 6 X_2 \leq 3600$;

$5(350) + 6(350) = 3850 > 3600$ Constraint is not satisfied.

C_4; $X_1 + 2 X_2 \leq 960$;

$350 + 2(350) = 1050 > 960$ Constraint is not satisfied.

Wilson does not have enough materials or the time necessary to manufacture according to this objective. Constraints C_3 and C_4 are infeasible points.

Fore any interior point there is always some distance from the constraints which is proportional to slack for ≤ and surplus for ≥ constraints (making RHS value non-negative) that prohibits the optimal solution until it has been removed. The second solution lies outside the feasibility region and therefore is not possible under the constraints imposed.

b. Wilson estimates that its profit is $7.00 per dozen baseballs and $10.00 per dozen softballs, the production schedule that maximizes their daily profit is found at the extreme point, (360, and 300).

$7(360) + 10(300) = 5,520$.

c. C_1; $X_1 \leq 500$ is a non-binding constraint

C_2: $X_2 \leq 500$ does not eliminate any points from consideration. It is redundant.

C_3; $5 X_1 + 6 X_2 \leq 3600$ is a binding constraint.

C_4; $X_1 + 2X_2 \leq 960$ is a binding constraint.

C_5; $Xj \geq 0$, $j = 1, 2$, are non-binding.

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