A Study on the Application of Fuzzy Inventory Models without allowing Storage Constraint

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Abstract: Application of inventory management problems compute the economic order quantity (EOQ) as a function of the setup cost and the holding cost in the deterministic model to help the decision maker of any business concern. A traditional way of approach will never allow fluctuations in these costs. Practically vagueness caused by the variation in fixing these costs is inevitable. In this paper parametric programming technique is utilized to compute EOQ, under fuzzy environment. Numerical illustrations are presented by considering basic inventory models.

Keywords: Inventory Management - Economical Order Quantity – Deterministic Model- Parametric Programming – Fuzzy Environment.

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1 INTRODUCTION

In this paper, the basic inventory models like instantaneous stock replenishment allowing no shortages, uniform stock replenishment without shortages, instantaneous replenishment allowing shortages are analysed in a fuzzy environment.

In the inventory management problems, under the deterministic type, the economic order quantity Q is considered as a function of setup cost (c_s) and holding cost (c_1). A traditional way of approach does not allow fluctuation in fixing these costs.

The time between the proposed day to start the production (or purchase) and the actual day of the commencement of the process causes vagueness in fixing these costs. Some basic inventory models such as instantaneous stock replenishment, uniform stock replenishment, allowing shortages are analyzed by assigning fuzzy quantities to these costs c_s and c_1 instead of crisp values. Parametric programming technique is applied to find the optimum value Q^o, and hence $TC_A{}^o$, the optimum total annual cost. Numerical Illustration is also given to each model

Inventory management requires demand forecasts as well as parameters for the inventory related cost, such as, carrying, replenishment, shortages, break-orders. Precise estimates of each of these quantity attributes is often difficult. Similarly, in production and process plan section problems, imprecision exists in specifying demand forecasts, inventory and processing cost parameters and processing times. When the problem is formulated with multiple objectives, the ambiguity is increased further. An early work using fuzzy concept in decision making has been performed by Bellman and Zedeh[1], through introducing fuzzy goals, costs and constraints. Lee et al [12] introduced the application of fuzzy set theory to lot-sizing in material requirements planning. In their paper, uncertainty in demand is modeled by using triangular fuzzy numbers. T.K.Roy and N.Maiti[13] presented an EOQ (Economic Order Quantity) model with constraint in a fuzzy environment. The model was solved by fuzzy non-linear programming method (FNLP), using Lagrange multipliers. K.Dhanam and A.Srinivasan[5] studied a simple item determination EOQ model in a fuzzy environment. The fuzzy goal and storage area have been represented by the hyperbolic membership functions and cost by linear membership function.

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Elementary inventory models, assuming instantaneous stock replenishment, uniform stock replenishment, and allowing shortages are analyzed. Numerical illustration is given for each of these models. Using the defuzzification technique [2, 8, 9, 17], Q^o and $TC_A{}^o$ are found and are compared with the corresponding crisp values. Vagueness in estimating setup cost, holding cost and shortage cost are modeled by trapezoidal membership function. Parametric programming technique is used to determine the optimum economic order quantity Q^o and the total annual cost($TC_A{}^o$). D.Jayaram and K.Kadambavanam [6], and K.Kadambavanam and P.Muthuswamy [7] have used the parametric programming technique in analyzing some advanced queuing models.

2 FORMULATION OF PARAMETRIC PROGRAMMING PROBLEM FOR SOME ELEMENTARY MODELS

The setup cost $\tilde{X}$, holding cost $\tilde{Y}_1$, and shortage cost $\tilde{Y}_2$ are approximately known and are represented by the following fuzzy sets:

$$\tilde{X} = \{(x, \mu_{\tilde{X}}(x) \mid x \in X)\} \qquad (2.1)$$

$$\tilde{Y}_i = \{(y_i, \mu_{\tilde{Y}_i}(y_i)) \mid y_i \in Y_i), (i = 1,2) \qquad (2.2)$$

where X, Y_1 and Y_2 are crisp universal sets of setup cost, holding cost and shortage cost respectively and $\mu_{\tilde{X}}(x)$ and $\mu_{\tilde{Y}_i}(y_i), (i = 1,2)$ are the respective membership functions. The α-cut of $\tilde{X}, \tilde{Y}_i, (i = 1,2)$, [4, 17], are

$$\alpha_{\tilde{X}} = \{(x \in X \mid \mu_{\tilde{X}}(x) \geq \alpha), \qquad (2.3)$$

$$\alpha_{\tilde{Y}_i} = \{(y_i \in Y_i \mid \mu_{\tilde{Y}_i}(y), \qquad (2.4)$$

where $0 < \alpha \leq 1$. The quantities $\alpha_{\tilde{X}}$ *and* $\alpha_{\tilde{Y}_i}, (i = 1,2)$ are crisp sets. Using α-cut the setup cost, holding cost and shortage cost can be represented by different levels of confidence intervals [4, 11, 15]. Hence the fuzzy inventory models could be reduced to a family of crisp inventory models, with different α-level cuts $\{\alpha_{\tilde{X}} \mid 0 < \alpha \leq 1\}$ and $\{\alpha_{\tilde{Y}_i} \mid 0 < \alpha \leq 1\}$, $(i = 1,2)$. $\alpha_{\tilde{X}}$ and $\alpha_{\tilde{Y}_i}, (i = 1, 2)$ are also denoted by $X(\alpha)$ and $Y_i(\alpha), (i = 1, 2)$ respectively. The above sets represent sets of movable boundaries and they form nested structure for expressing the relationship between the crisp sets and fuzzy sets.

Let the confidence intervals of the fuzzy sets $\tilde{X}$ and $\tilde{Y}_i$, $(i = 1,2)$ be $[l_{x(\alpha)}, u_{x(\alpha)}]$ and $[l_{y_i(\alpha)}, u_{y_i(\alpha)}]$, $(i = 1,2)$ respectively. Since the set-up cost, holding cost and shortage cost are fuzzy numbers, using Zadeh's extension principle [12, 17], the membership function of the performance measure $p(\tilde{X}$ and $\tilde{Y}_i)$, $(i = 1,2)$ is defined as

$$\mu_{p(\tilde{X},\tilde{Y}_i)}(z) = \sup_{x \in \tilde{X}, y_i \in \tilde{Y}_i} \min\{\mu_{\tilde{X}}(x), \mu_{\tilde{Y}_i}(y_i) / z = p(x, y_i)\}, (i = 1,2) \qquad (2.5)$$

Construction of the membership function $\mu_{p(\tilde{X},\tilde{Y}_i)}(z)$, $(i = 1,2)$, is equivalent to the derivation of α cuts of $\mu_{p(\tilde{X},\tilde{Y}_i)}(z)$, $(i = 1,2)$.

From the equation (2.5) the relation $\mu_{p(\tilde{X},\tilde{Y}_i)}(z) = \alpha$, $(i = 1,2)$ is true only when $\mu_{\tilde{X}}(x) = \alpha$,

$\mu_{\tilde{Y}_i}(y_i) \geq \alpha$ or $\mu_{\tilde{X}}(x) \geq \alpha$, $\mu_{\tilde{Y}_i}(y_i) = \alpha$ is true.

The parametric programming problems have the following form:

$$l_{p(\alpha)} = \min p(x, y_i) \qquad (2.6)$$

such that

$$l_{x(\alpha)} \leq x \leq u_{x(\alpha)},$$

$$l_{y_i(\alpha)} \leq y_i \leq u_{y_i(\alpha)}, (i = 1,2),$$

and

$$u_{p(\alpha)} = \max p(x, y_i) \qquad (2.7)$$

such that

$$l_{x(\alpha)} \leq x \leq u_{x(\alpha)},$$

$$l_{y_i(\alpha)} \leq y_i \leq u_{y_i(\alpha)}, (i = 1,2),$$

If both $l_{p(\alpha)}$ *and* $u_{p(\alpha)}$ are invertible with respect to α then the left shape function

$L(z) = l^{-1}p(a)$ and the right shape function $R(z) = u^{-1}p(a)$ [2, 15] can be obtained. From this

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the membership function $\mu_{p(\tilde{X},\tilde{Y}_i)}(z)$, $(i = 1,2)$ is constructed as

$$\mu_{p(\tilde{X},\tilde{Y}_i)}(z)=\begin{cases} L(z) & for\ z_1 \le z \le z_2 \\ 1 & for\ z_2 \le z \le z_3 \\ R(z) & for\ z_3 \le z \le z_4 \end{cases} \qquad (2.8)$$

where $z_1 \le z_2 \le z_3 \le z_4$, $L(z_1) = R(z_4) = 0\ and\ L(z_2) = R(z_3) = 1$.

3 ELEMENTARY INVENTORY MODELS

3.1 MODEL-I. EOQ problems with instantaneous replenishment and no shortages

The objective of this study is to determine the optimum order quantity (EOQ) such that the total inventory cost is minimized.

Assumptions

3.1.1 The inventory system pertains to a single item.

3.1.2 Annual Demand (D) is deterministic.

3.1.3 The inventory is replenished in a single delivery for each order.

3.1.4 Replenishment is instantaneous.

3.1.5 There is no lead time.

3.1.6 Shortages are not allowed.

Using the concept of α cut, the above fuzzy inventory model can be reduced as EOQ model with instantaneous replenishment and no shortage [10, 14, 16] for which

$$Q^0 = \left[\frac{2DC_s}{C_1}\right]^{\frac{1}{2}} \qquad (3.1.1)$$

and

$$TC_A^0 = \left[\frac{Q^0 C_1}{2}\right] + \left[\frac{DC_s}{Q^0}\right] = [2DC_sC_1]^{\frac{1}{2}} \qquad (3.1.2)$$

where $C_s\ and\ C_1$ represent the set-up cost and the holding cost respectively.

3.2 MODEL-II. EOQ problems with uniform replenishment and no shortages

Assumptions

3.2.1 The inventory system pertains to a single item.

3.2.2 Annual Demand (D) is deterministic.

3.2.3 The rate of replacement k in inventory is finite.

3.2.4 The requirement (sales) or the decreasing rate r of inventory per unit of time is finite (k >r).

3.2.5 Each production run is split into two parts t_1 and t_2 (i.e., $t = t_1 + t_2$). During t_1 , the inventory is building up at a constant rate of (k – r) units, per unit of time; during t_2, no replenishment takes place and the inventory level decreases at the rate of r per unit time.

3.2.6 There is no lead time.

3.2.7 Shortages are not allowed.

sing the concept of α cut for the membership function given in equation (2.8) it can be reduced as EOQ model with uniform replenishment and no shortages [10, 14, 16] for which

$$Q^0 = \left[\left(\frac{2DC_s}{C_1}\right)\left(\frac{K}{K-r}\right)\right]^{\frac{1}{2}} \qquad (3.2.1)$$

and

$$TC_A^0 = \frac{DC_s}{Q^0} + \frac{Q^0}{2}\left(1 - \frac{r}{k}\right)C_1 = \left[2DC_sC_1\left(1 - \frac{r}{k}\right)\right]^{\frac{1}{2}} \qquad (3.2.2)$$

3.3 MODEL-III. EOQ problems with instantaneous replenishment with shortages

Assumptions

3.3.1 All the assumptions from (3.1.1) to (3.1.5) are carried forward here also.

3.3.2 Shortages are allowed and C_2 is shortage cost for unit of item.

Using the concept of α cut, for the membership function obtained from the equation (2.8) it can be reduced as EOQ model with instantaneous replenishment and shortages [10, 14], for which, the optimum stock level

$$Q^0 = \left[\left(\frac{2DC_s}{C_1}\right)\left(\frac{C_2}{C_1+C_2}\right)\right]^{\frac{1}{2}} \quad (3.3.1)$$

and

$$TC_A^0 = \left[2DC_sC_1\left(\frac{C_2}{C_1+C_2}\right)\right]^{\frac{1}{2}} \quad (3.3.2)$$

4. ILLUSTRATIONS

4.1 Model – 1 EOQ problem with instantaneous replenishment allowing no shortages.

The set cost and holding cost are fuzzy numbers represented by $\overline{X}$ =[200,250,350,400] and $\overline{Y}_1$ =[0.6,0.7,1.0,1.1]. The α-cut of the membership function $\mu_{\overline{X}}(x)$, $\mu_{\overline{Yi}}(yi)$)are [200+50α,400-50α] and [0.6+0.1α,1.1-0.1 α] respectively. From the equation (2.6) and (2.7), the parametric programming problems are formulated to derive the membership function for $\overline{Q}^o$

They are of the form

$$l_{Qo(\alpha)} = \min\left\{\frac{2Dx}{y_1}\right\}^{1/2}$$

With $200+50\alpha \le x \le 400-50\alpha$

$0.6+0.1\alpha \le y_1 \le 1.1-0.1\alpha$ (4.1.1)

and $u_{Q^o(\alpha)} = \max\left\{\frac{2Dx}{y_1}\right\}^{1/2}$

With $200+50\alpha \le x \le 400-50\alpha$

$0.6+0.1\alpha \le y_1 \le 1.1-0.1\alpha$ (4.1.2)

Where o<α≤1

$l_{Qo(\alpha)}$ is found when x and y_1 approach their lower and higher bound respectively. Taking D=20000 units, the optimal solution for (4.1.1) is

$$l_{qo(\alpha)} = \left[\frac{4\times10^5\times(200+50\alpha)}{11-\alpha}\right]^{1/2} \quad (4.1.3)$$

Also $u_{Q^o(\alpha)}$ is found when x and y_1 approach their upper bound and lower bound respectively. In this case the optimal solution for (4.1.2) is

$$u_{Qo(\alpha)} = \left[\frac{4\times10^5\times(400-50\alpha)}{6+\alpha}\right]^{1/2} \quad (4.1.4)$$

The membership function $\mu_{Q^o(z)}$ is obtained and given by

$$\mu_{\overline{Q^o(z)}} = \begin{cases} \dfrac{11z^2-8\times10^7}{z^2+2\times10^7} & for\ 2696.7995 \le z \le 3162.2777 \\ 1 & for\ 3162.2777 \le z \le 4472.1360 \\ \dfrac{16\times10^7-6z^2}{2\times10^7+z^2} & for\ 4472.1360 \le z \le 5163.9778 \end{cases}$$

(4.1.5)

The graph of $\mu_{\overline{Q^o(z)}}$ is depicter in fig (1) The parametric programming problem corresponding to the total annual cost TC^o has different only from the (4.1.1) and (4.1.2) in the objective function and given below

$l_{TC^o}(\alpha)$ = min $\{2Dxy_1\}^{1/2}$ (4.1.6)

And $u_{TC^o}(\alpha)$ =max $\{2Dxy_1\}^{1/2}$ (4.1.7)

From the above problem (4.1.6) & (4.1.2) $l_{TC^o}(\alpha)$ is obtained when both x and y_1 approach their lower bound $uTC^o(\alpha)$ is obtained when both x and y_1 approach their upper bond.

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$$l_{TC^o}(\alpha)=[4\times10^4\times(120+50\alpha+5\alpha^2)]^{\frac{1}{2}}$$
(4.1.8)

$$u_{TC^o}(\alpha)=[4\times10^4\times(440-95\alpha+5\alpha^2)]^{\frac{1}{2}}$$
(4.1.9)

The membership function $\mu_{TC^o(Z)}$ is given below

$$\mu_{TC^o(Z)} = \begin{cases} \dfrac{-100\times10^2+\sqrt{4\times10^6+20z^2}}{2\times10^3} & for\ 2190.8902\le z\le 2645.7513 \\ 1 & for\ 2645.7513\le z\le 3741.6574 \\ \dfrac{190\times10^2-\sqrt{9\times10^6+20z^2}}{2\times10^3} & for\ 3741.6574\le z\le 4195.2354 \end{cases}$$
(4.1.10)

The graph of $\mu_{\overline{TC^o(z)}}$ is given in fig(2). For the crisp values cs=Rs.500, C_1=0.9, D=20,000 units

Q^o = 4714.0452 and TC^o = Rs. 4242.6407

But defuzzification [6] of $\mu_{\overline{Q^o(z)}}$ and $\mu_{\overline{TC^o(z)}}$ give Q^o = 3817.2069 and TC^o =3193.7044 respectively.

Therefore the value of TC^o for the corresponding value of Q^o =4714.0452 is 3944.0531.

So if definite fluctuation of ₹ 50/- and the maximum possible fluctuation of ₹ 100/- in C_s and definite fluctuation of ₹ 0.1 and the maximum possible fluctuation of ₹ 0.3 in C_1 are identified, then the amount of Q^o to be increase suitably (it can be computed by using the above technique). So as to maintain the total annual cost.

4.2 Model – II: EOQ problems with uniform replenishment without shortage:

In this case , the fuzzy number $\overline{X}$ *and* $\overline{Y}_1$ for the set up cost and the holding cost are as defined as in model I . The parametric programming problems are differed only in the objective functions are listed below

$$l_{Qo(\alpha)}=\min\left\{\left(\frac{2Dx}{y_1}\right)\left(\frac{k}{k-r}\right)\right\}^{1/2} \quad (4.2.1)$$

$$and\ u_{Qo(\alpha)}=\max\left\{\left(\frac{2Dx}{y_1}\right)\left(\frac{k}{k-r}\right)\right\}^{1/2} \quad (4.2.2)$$

The optimal solutions to the problems (4.2.1) and (4.2.2) are

$$l_{Qo(\alpha)}=\left[\left(\frac{10^3\times D\times(4+\alpha)}{11-\alpha}\right)\left(\frac{k}{k-r}\right)\right]^{\frac{1}{2}}\ and \quad (4.2.3)$$

$$u_{uo(\alpha)}=\left[\left(\frac{10^3\times D\times(8-\alpha)}{6+\alpha}\right)\left(\frac{k}{k-r}\right)\right]^{\frac{1}{2}} \quad (4.2.4)$$

Taking k = 100 units, r = 50 units and 300 working days per year, the membership function $\mu_{\overline{Q^o(z)}}$ has the following form

$$\mu_{\overline{Q^o(z)}} = \begin{cases} \dfrac{275z^2-3\times10^9}{75\times10^7+25z^2} & for\ 3302.8913\le z\le 3872.9833 \\ 1 & for\ 3872.9833\le z\le 5477.2256 \\ \dfrac{6\times10^9-150z^2}{75x10^7+25z^2} & for\ 5477.2256\le z\le 6324.5553 \end{cases}$$
(4.2.5)

The graph of $\mu_{\overline{Q^o(z)}}$ is given in fig(3).

The membership function $\mu_{\overline{TC^o(z)}}$ is obtained by solving the following problems (4.2.6) and (4.2.7).

$$l_{TC^o(\alpha)}=\min\left\{(2Dxy_1)(1-\frac{r}{k})\right\}^{1/2}\ and \quad (4.2.6)$$

$$u_{TCo(\alpha)}=\max\left\{(2Dxy_1)(1-\frac{r}{k})\right\}^{1/2} \quad (4.2.7)$$

together with the constraints listed in problems (4.1.1) & (4.1.2).The resulting membership function $\mu_{\overline{TC^o(z)}}$ is

$$\mu_{\overline{TC^o(z)}} = \begin{cases} \dfrac{-50+\sqrt{10^2+(z^2/750)}}{10} & \text{for } 1341.6408 \le z \le 1620.1852 \\ 1 & \text{for } 1620.1852 \le z \le 2291.2878 \\ \dfrac{95-\sqrt{225+(z^2/750)}}{10} & \text{for } 2291.2878 \le z \le 2569.0465 \end{cases} \quad (4.2.8)$$

The graph of the membership function $\mu_{\overline{TC^o}}(z)$ is depicted fig (4). Defuzzification [6] of $\mu_{\overline{Q^o(z)}}$ and $\mu_{\overline{TC^o(z)}}$ give the value of Q^o and TC^o are respectively 4675.1045 units and ₹ 1955.7365, where as the corresponding crisp quantities are respectively 5773.5 units and ₹ 2598.76. For defuzzitication the total cost for purchasing 5773.5 units is ₹ 2104.3517. Thus in this method also the fluctuation in the C_s and C_1, the EOQ Q^o is suitable increased to the total minimum annual cost as in the case of non – fluctuated situation.

4.3 Model – III . EOQ problem with instantaneous replenishment and shortages

For this method, the fuzzy numbers $\overline{X}$ and $\overline{Y}_1$ for the setup cost and the holding cost are defined as in model1. The fuzzy number for the shortages is taken as $\bar{y}_2 = [0.7, 0.8, 1.1, 1.2]$ and the corresponding α cut of the membership function is $[0.7+0.1\alpha, 1.2-0.1\alpha]$ .The parametric programming problems to determine $\mu_{\overline{Qo}}(z)$ are formulated as follows:

$$l_{Qo(\alpha)} = \min\left\{\left(\frac{2Dx}{y_1}\right)\left(\frac{1}{\left(\frac{y_1}{y_2}\right)+1}\right)\right\}^{\frac{1}{2}} \quad (4.3.1)$$

with

$$200+50\alpha \le x \le 400-50\alpha$$

$$0.6+0.1\alpha \le y_1 \le 1.1-0.1\alpha$$

$$0.7+0.1\alpha \le y_2 \le 1.2-0.1\alpha$$

and

$$u_{Qo(\alpha)} = \max\left\{\left(\frac{2Dx}{y_1}\right)\left(\frac{1}{\left(\frac{y_1}{y_2}\right)+1}\right)\right\}^{\frac{1}{2}} \quad (4.3.2)$$

with

$$200+50\alpha \le x \le 400-50\alpha$$

$$0.6+0.1\alpha \le y_1 \le 1.1-0.1\alpha$$

$$0.7+0.1\alpha \le y_2 \le 1.2-0.1\alpha$$

Assuming D=20000 units, the optimum solution for the problem (4.3.1) and (4.3.2) are obtained and given by

$$l_{Qo(\alpha)} = \left[\frac{2\times 2\times 10^4 \times (5\alpha^2+55\alpha+140)}{1.98-0.18\alpha}\right]^{1/2} \quad (4.3.3)$$

$$\text{and } u_{Q^o(\alpha)} = \left[\frac{2\times 2\times 10^4 \times (5\alpha^2-100\alpha+480)}{1.08+0.18\alpha}\right]^{1/2} \quad (4.3.4)$$

The corresponding membership function $\mu_{\overline{Q^o(z)}}$ has the following form:

$$\mu_{\overline{Q^o(z)}} = \begin{cases} \dfrac{-\left(55+\dfrac{0.18z^2}{4\times10^4}\right)+\sqrt{\left(55+\dfrac{0.18}{4\times10^4}.z^2\right)^2-20\left(140-\dfrac{1.98z^2}{4\times10^4}\right)}}{10} & for\ 1681.7499\le z\le 2108.1851 \\ 1 & for\ 2108.1851\le z\le 3496.0295 \\ \dfrac{\left(100+\dfrac{0.18z^2}{4\times10^4}\right)-\sqrt{\left(100+\dfrac{0.18z^2}{4\times10^4}\right)^2-20\left(480-\dfrac{1.08z^2}{4\times10^4}\right)}}{10} & for\ 3496.0295\le z\le 4216.3702 \end{cases} \quad (4.3.5)$$

The membership function $\mu_{\overline{Q^o(z)}}$ is depicter fig(5).The objective functions of the parametric programming problem to determine $\mu_{\overline{TC^o(z)}}$ are

$$l_{TC^o(\alpha)} = \min\left\{(2Dxy_1)\left(\frac{1}{\left(\frac{y_1}{y_2}\right)+1}\right)\right\}^{\frac{1}{2}} \quad (4.3.6)$$

and

$$u_{TC^o(\alpha)} = \max\left\{(2Dxy_1)\left(\frac{1}{\left(\frac{y_1}{y_2}\right)+1}\right)\right\}^{\frac{1}{2}} \quad (4.3.7)$$

The constraint part of the problem (4.3.5) &(4.3.7) are as same as problem of (4.3.1).

$$l_{\overline{TC^o(\alpha)}} = \left[\frac{2\times2\times10^4\times(84+47\alpha+8.5\alpha^2+.5\alpha^3}{1.3+0.2\alpha}\right]^{\frac{1}{2}} \quad (4.3.8)$$

$$and\ \ u_{\overline{TC^o(\alpha)}} = \left[\frac{2\times2\times10^4\times(528-158\alpha+15.5\alpha^2-0.5\alpha^3}{2.3-0.2\alpha}\right]^{\frac{1}{2}} \quad (4.3.9)$$

Neglect higher powers of α, we get

$$l_{\overline{TC^o(\alpha)}} = \left[\frac{2\times2\times10^4\times(84+47\alpha+8.5\alpha^2}{1.3+0.2\alpha}\right]^{\frac{1}{2}}$$

$$and\ \ u_{\overline{TC^o(\alpha)}} = \left[\frac{2\times2\times10^4\times(528-158\alpha+15.5\alpha^2}{2.3-0.2\alpha}\right]^{\frac{1}{2}}$$

The corresponding membership function $\mu_{\overline{TC^o(\alpha)}}$ has the following form:

$$\mu_{\overline{TC^o(z)}} = \begin{cases} \dfrac{-\left(47-\dfrac{0.2z^2}{2\times2\times10^4}\right)+\sqrt{\left(47-\dfrac{0.2z^2}{2\times2\times10^4}\right)^2-34\left(84-\dfrac{1.3z^2}{2\times2\times10^4}\right)}}{17} & for\ 1607.6739\le z\le 1928.7302 \\ 1 & for\ 1928.7302\le z\le 2709.7707 \\ \dfrac{\left(158-\dfrac{0.2z^2}{2\times2\times10^4}\right)-\sqrt{\left(158-\dfrac{0.2z^2}{2\times2\times10^4}\right)^2-62\left(528-\dfrac{2.3z^2}{4\times10^4}\right)}}{31} & for\ 2709.7707\le z\le 3030.2820 \end{cases}$$

The corresponding graph of the membership function $\mu_{\overline{TC^o(z)}}$ is given in fig(6). When $y_2 = \infty$ (i.e. when the shortages are not allowed) from the objective function given by the equation through (3.3.1) to (3.3.7) it is evident that model III becomes model I.

Defuzzification of the membership function $\mu_{\overline{Q^o(z)}}$ *and* $\mu_{\overline{TC^o(z)}}$ and respectively 2802.1073 units are ₹ 2319.2505, where as the corresponding crisp quantities are respectively 3419.9278 units are ₹ 3077.9351.

Comparison Table

S.No	Model	Fuzzy Quantity		Crisp Quantity		Percentage of Variation in Fuzzy Quantity
		Q^0	TC^0	Q	TC	

1	I	3817.2069	3193.7044	4714.0452	4242.6407	7.5706
2	II	4675.1045	1955.7365	5773.5	2598.76	7.5989
3	III	2802.1073	2319.2505	3419.9278	3077.9351	8.7376

Conclusion

In this paper, the membership functions for the economic order quantity Q^o and the total cost TC^o are obtained for some elementary fuzzy inventory models. In all the given elementary models, the total costs corresponding to the fuzzy models are lesser than the total costs corresponding to the crisp models. From this we conclude that, the fuzzy models are more convenient than the crisp models.

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Fig 1

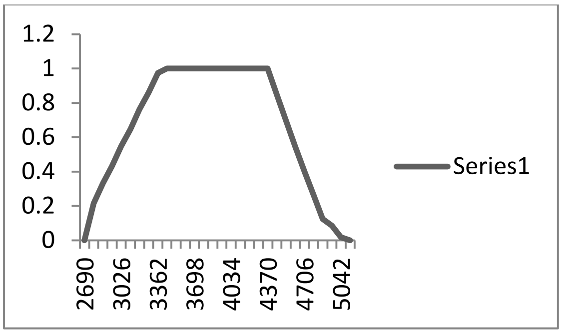
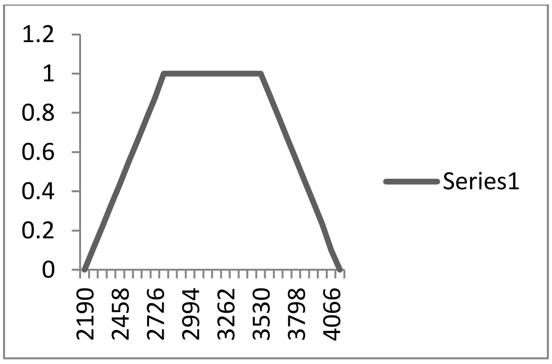


Fig 2



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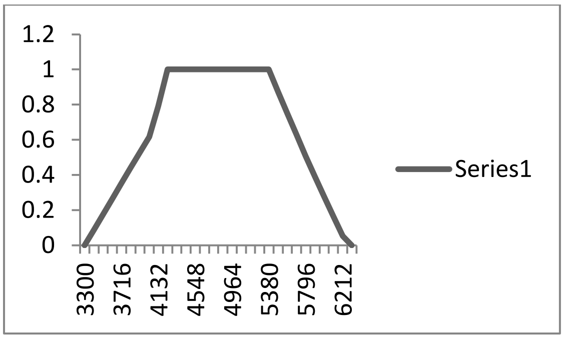

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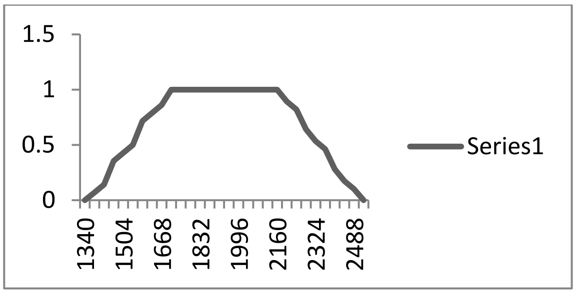

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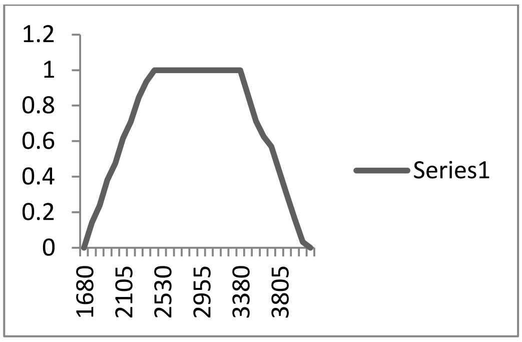

Fig 6